Analysis of Adaptive Detection of Partially-Correlated χ^2 Targets in Multitarget Environments

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Abstract

The target's characteristics play an important role in the design and performance analysis of all radar systems. The fluctuation rate of such a target may vary from essentially independent return amplitudes from pulse-to-pulse to significant variation only on a scan-to-scan basis. The correlation coefficient between the two consecutive echoes in the dwell-time is equal to unity for the SWI and SWIII cases while it is zero for the SWII and SWIV models. An important class of targets is that represented by the so-called moderately fluctuating Rayleigh and χ^2 targets. The detection of partially correlated χ^2 targets with two and four degrees of freedom is therefore of great importance.

Our goal in this paper is to analyze the detection performance of the most familiar candidates of CFAR techniques for the case where the radar receiver post-detection integrates M pulses of an exponentially correlated signal from targets which exhibit χ^2 statistics with two and four degrees of freedom. Exact formulas for the detection probabilities are derived, in the absence as well as in the presence of spurious targets. As predicted, the CA detector has the best homogeneous performance while the OS scheme gives the best target multiplicity performance when the number of outlying targets is within its allowable values.

Keywords

Post-detection Integration; Partially Correlated χ^2 Targets; Swerling Fluctuation Models; CFAR Detection Techniques; Target Multiplicity Environments

I. Introduction

The complete radar signal over a multiple observation interval is called a pulse train. Fluctuating pulse trains occur often in practice. When a radar target consists of several relatively strong reflecting surfaces displaced from one another by the order of a wavelength, the amplitude and phase of the composite radar echo are sensitive to the spatial orientation of the target. Moreover, if the target has relative motion with respect to the radar, it presents a time varying radar cross section. This change may be a slow variation and occur on a scan-to-scan basis (on successive antenna scans across a target) or on a pulse-to-pulse basis (on successive sweeps). Because it is difficult to predict the exact nature of the change, a statistical description is often adopted to characterize the target radar cross section [Meyer, 1973].

Target fluctuation is defined as variation in the amplitude of a target signal, caused by changes in target aspect angle, rotation, vibration of target scattering sources, or changes in radar wavelength. The fluctuation rate of a radar target may vary from essentially independent return amplitudes from pulse-to-pulse to significant variation only on a scan-to-scan basis. Three families of radar cross-section fluctuation models have been used to characterize most target populations of interest: the χ^2 family, the Rice family, and the log-normal family [Aloisio, 1994]. The χ^2 family includes the Rayleigh (SWI & SWII) model with two degree of freedom (κ=1), the four degree of freedom (κ=2) model (SWIII & SWIV), the Weinstock model (κ<1), and the generalized model (κ a positive real number). The χ^2 models are used to represent complex targets such as aircraft and have the characteristic that the distribution is more concentrated about the mean as the value of the parameter κ is increased [Swerling, 1997]. The advantages of the Swerling target models are that they bracket a large number of real target classes [Di Vito, 1999].

It is often assumed that the Swerling cases bracket the behavior of fluctuating targets of practical interest. However, recent investigations of target cross section fluctuation statistics indicate that some targets may have probability of detection curves which lie considerably outside the range of cases which are satisfactorily bracketed by the Swerling cases. An important class of targets is represented by the so-

called moderately fluctuating Rayleigh and χ^2 targets [El_Mashade, 2011] which when illuminated by a coherent pulse train, return a train of correlated pulses with a correlation coefficient in the range 0<ρ<1. The detection of this type of fluctuating targets is therefore of great interest [Hou, 1987].

The constant false alarm rate (CFAR) detection techniques represent the backbone upon which modern radar systems investigate in their operation. These processors make use of the fact that the amplitude variation of weather and sea clutter has a Rayleigh distribution, as well as the capability to reduce the clutter output to about the same level as the receiver noise level. As a consequence, much attention has been paid to the task of designing and assessing these adaptive detection techniques. The CA- and OS-CFAR detectors are the most widely used ones in the CFAR world [[El_Mashade, 2008]. While the performance of the CA detector is optimum in homogeneous situations, this performance degrades rapidly in non-ideal conditions caused by multiple target and non-uniform clutter. The OS processor, on the other hand, a CFAR technique, is relatively immune to non-homogeneous cases resulted from outlying targets and clutter edges. This technique relies on the order of the samples in the reference window and takes an appropriate reference cell to estimate the clutter power level. The OS trades a small loss in detection performance, relative to the CA, in ideal conditions for mush less performance degradation in non-ideal conditions.

This paper is devoted to the analysis on the performance of CA- and OS-CFAR detectors for partially-correlated χ^2 targets with two and four degrees of freedom in the absence as well as in the presence of spurious targets. In section II, we formulate the problem and compute the characteristic function of the post-detection integrator output for the case where the signal fluctuation obeys χ^2 statistics with two and four degrees of freedom. The ideal performance of the schemes under consideration is analyzed in section III. Section IV deals with the problem of multiple-target environment and its effect on the behavior of the CA and OS processors. In section V, a brief discussion is made along with our conclusions.

II. Theoretical Background and Statistical Model

Let the input target signal and noise to the square-law detector be represented by the complex vectors u + jv and a + jb, respectively. u and v represent the in-phase and quadrature components of the target signal at the square-law detector, while a and b represent the in-phase and quadrature components of the noise, respectively. The target is assumed to be independent of the noise and the in-phase samples to be independent and identically distributed (IID) with the Gaussian probability density function (PDF), while the target samples are supposed to be identically distributed but correlated. The output of M-pulse non-coherent integrator, normalized to the noise power, is

$$\nu \triangleq \frac{1}{2\psi} \sum_{\ell=1}^{M} \left(\left| u_\ell + a_\ell \right|^2 + \left| v_\ell + b_\ell \right|^2 \right) \tag{1}$$

Let the in-phase received target signal and noise vectors be U and A, respectively, where

$$U \triangleq \left[u_1, u_2, u_3, \ldots\ldots\ldots, u_M\right]^T \quad \text{and} \quad A \triangleq \left[a_1, a_2, a_3, \ldots\ldots\ldots, a_M\right]^T \tag{2}$$

The characteristic function (CF) of ν can be expressed as

$$C_\nu(\omega) \triangleq \int_{-\infty}^{\infty} p_\nu(y) \exp(-\omega y) \, dy = \left\{ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_U(u)\, p_A(a) \exp\left(-\frac{\omega}{2\psi} \sum_{i=1}^{M} \left| u_i + a_i \right|^2 \right) da\, du \right\}^2$$

(3)In the above expression, $p_U(u)$ and $p_A(a)$ denote the joint probability density functions (PDF's) of U and A, respectively. Based on our previous assumptions, one can write the joint PDF of A as

$$p_A(a) = \frac{1}{(2\pi\psi)^{M/2}} \exp\left(-\frac{1}{2\psi} \sum_{j=1}^{M} a_j^2 \right) \tag{4}$$

The substitution of Eq.(4) in Eq.(3) yields

$$C_\nu(\omega) = \left\{ \left(\frac{1}{2\pi\psi} \right)^{M/2} \int_{-\infty}^{\infty} p_U(u)\, du \int_{-\infty}^{\infty} \exp\left[-\frac{1}{2\psi} \sum_{i=1}^{M} \left(\omega \left| u_i + a_i \right|^2 + \left| a_i \right|^2 \right) \right] da \right\}^2 = \frac{1}{(\omega+1)^M} \left\{ \int_{-\infty}^{\infty} p_U(u) \exp\left(-\frac{\omega}{2\psi(\omega+1)} \sum_{j=1}^{M} \left| u_j \right|^2 \right) du \right\}^2 \tag{5}$$

There are many probability density functions for target cross section which are used to characterize fluctuating targets. The more important PDF is the so-called χ^2 distribution with 2κ degrees of freedom. The χ^2 family includes the Rayleigh (SWI & SWII) model, the four-degree of freedom model (SWIII & SWIV), the Weinstock model (κ <1) and the generalized model (κ a positive real number). These models used to represent complex targets such as aircraft have the characteristic that the distribution is more concentrated about the mean as the value of the parameter κ is increased. The χ^2- distribution with 2κ degrees of freedom is given by

$$p\left(\sigma/\bar{\sigma}\right) = \frac{1}{\Gamma(\kappa)} \left(\frac{\kappa}{\bar{\sigma}} \right)^{\kappa} \sigma^{\kappa-1} \exp\left(-\frac{\kappa}{\bar{\sigma}} \sigma \right) U(\sigma) \tag{6}$$

$\bar{\sigma}$ stands for the average cross section over all target fluctuations and U(.) denotes unit step function. When κ=1, the PDF of Eq.(6) reduces to the exponential or Rayleigh power distribution that is applied to the Swerling case I. Swerling cases II, III, and IV correspond to κ=M, 2, and 2M, respectively. When κ tends to infinity, the χ^2- distribution corresponds to the non-fluctuating target. It is of importance to note that the χ^2- distribution with 2κ degrees of freedom can be obtained by adding squared magnitude of κ complex Gaussian random variables.

II-1 χ^2-Distribtion with Two-degrees of Freedom

If κ=1, then σ may be generated as $\sigma=w_1^2+w_2^2$, where w_i's are independent and identically distributed (IID) Gaussian random variables, each with zero mean and $\bar{\sigma}/2$ variance. The magnitude of the in-phase component u ($u=|w_1|$) has a PDF given by

$$p_U(u)\,du = p_{w_1}(w_1)\,dw_1 = \frac{1}{\sqrt{2\pi\alpha}} \exp\left(-\frac{w_1^2}{2\alpha}\right) dw_1 \quad with \quad \alpha \triangleq \frac{\bar{\sigma}}{2} \tag{7}$$

To accommodate an Mx1 vector of correlated χ^2 RV's with two degrees of freedom, the PDF of the M-dimensional vector W_1 is inreoduced. Therefore, the joint PDF of u_i's, i=1. 2, ….. , M, has a form given by

$$\begin{aligned} p_U(u)\;du &= p_{w_1}(w_1)\;dw_1 \\ &= \left(\frac{1}{2\pi\alpha}\right)^{M/2} \frac{1}{|\Lambda|^{1/2}} \exp\left(-\frac{w_1^T\Lambda^{-1}w_1}{2\alpha}\right) dw_1 \end{aligned} \tag{8}$$

Λ is the correlation matrix of u_1, u_2, …., u_M and T denotes transposition. As **I** denotes the identity matrix, the substitution of Eq.(8) into Eq.(5) yields

$$C_\nu(\omega) = \left(\frac{1}{\omega+1}\right)^M * \left\{\int_{-\infty}^{\infty} \frac{1}{(2\pi\alpha)^{M/2}\sqrt{|\Lambda|}} \exp\left[-\frac{1}{2\alpha}\left(w_1^T\{\Lambda^{-1}\}+\frac{\alpha}{\psi}\frac{\omega}{\omega+1}I\right)w_1\right]dw_1\right\}^2 \tag{9}$$

If the average signal-to-noise ratio (SNR) is assumed to be $\alpha/\psi=\Omega$, Eq.(9) can be re-expressed as

$$C_\nu(\omega) = \prod_{\ell=1}^{M} \frac{1}{1+(1+\lambda_\ell\Omega)\omega} \tag{10}$$

λ_i's are the nonnegative eigenvalues of Λ. The CF of Eq.(10) for the SWI target fluctuation model is represented by choosing λ_1=M, λ_i's=0, i=2, 3, …, M, and that for the SWII case can be modeled by the assumption that λ_i's=1, i=1, 2, …, M. For the CFAR processor performance analysis to be simple, the above equation can be put in another clarified form as

$$C_\nu(\omega) = \prod_{j=1}^{M} \frac{d_j}{\omega+d_j} \quad \& \quad d_j \triangleq \frac{1}{1+\Omega\lambda_j} \tag{11}$$

II-2 χ^2-Distribtion with Four-degrees of Freedom

If κ=2, then σ and the squared magnitude of its in-phase component, u, may be generated as follows:

Let w_i's, i=1, …, 4, be IID Gaussian random variables, each one is of zero mean and of $\alpha=\bar{\sigma}/4$ variance, σ and $|u|$ are given by [Weiner, 1988]:

$$\sigma = \sum_{\ell=1}^{4} w_\ell^2 \quad and \quad |u| \triangleq \sqrt{w_1^2+w_2^2} \tag{12}$$

Since w_1 and w_2 are IID, one may write

$$\begin{aligned} p_U(u)\,du &= p_{w_1}(w_1)\,p_{w_2}(w_2)\,dw_1\,dw_2 \\ &= \frac{1}{2\pi\alpha}\exp\left\{-\frac{w_1^2+w_2^2}{2\alpha}\right\} dw_1\,dw_2 \end{aligned} \tag{13}$$

Therefore, the joint PDF of u_1, u_2, ….., u_M is of the form

$$\begin{aligned} p_U(u)\,du &= p_{w_1}(w_1)\,p_{w_2}(w_2)\,dw_1 dw_2 \\ &= \left(\frac{1}{2\pi\alpha}\right)^M\left(\frac{1}{|\Lambda|}\right)\exp\left\{-\frac{w_1^T\Lambda^{-1}w_1+w_2^T\Lambda^{-1}w_2}{2\alpha}\right\} \\ &\quad dw_1\,dw_2 \end{aligned} \tag{14}$$

where Λ is the correlation matrix of $\mathbf{w}_1$ and $\mathbf{w}_2$. Note that $\mathbf{w}_1$ and $\mathbf{w}_2$ are uncorrelated, but components of $\mathbf{w}_1$ are correlated with each other, as the components of $\mathbf{w}_2$. Therefore, Eq.(5) can be rewritten as

$$\begin{aligned} C_\nu(\omega) = \left(\frac{1}{\omega+1}\right)^M \Bigg\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left(\frac{1}{2\pi\alpha}\right)^M\frac{1}{|\Lambda|} \; * \\ \exp\left[-\frac{1}{2\alpha}\left\{w_1^T\left(\Lambda^{-1}+\frac{\alpha}{\psi}\frac{\omega}{\omega+1}I\right)w_1\right\}\right] * \\ \exp\left[-\frac{1}{2\alpha}\left\{w_2^T\left(\Lambda^{-1}+\frac{\alpha}{\psi}\frac{\omega}{\omega+1}I\right)w_2\right\}\right] dw_1\,dw_2\Bigg\}^2 \end{aligned} \tag{15}$$

If the average SNR (2α/ψ) is set as Ω, the above equation can be re-expressed as

$$C_\nu(\omega) = \prod_{\ell=1}^{M} \frac{\omega+1}{\left\{\left(1+\Omega\frac{\lambda_\ell}{2}\right)\omega + 1\right\}^2} \tag{16}$$

The CF of Eq.(16) for the Swerling III target fluctuation model is represented by choosing λ_1=M, λ_i's=0, i=2, 3, …, M, and that for the Swerling IV case can be modeled by the assumption of λ_i's=1, i=1, 2, …, M. To simplify the detection performance evaluation of the CFAR processors, the above equation can be reformatted to have an easily mathematical form such as

$$C_\nu(\omega) = \prod_{j=1}^{M}(\omega+1)\left\{\frac{b_j}{\omega+b_j}\right\}^2 \quad with \quad b_j \triangleq \frac{1}{1+\Omega\lambda_j/2} \tag{17}$$

In view of Eqs.(11 & 17), the solutionto partially-correlated case requires the computation of the eigen-values of the correlation matrix Λ. It is assumed here that i) the statistics of the signal are stationary, and ii) the signal can be represented by a first order Markov process. Under these assumptions, Λ is a Toeplitz nonnegative definite matrix. Thus,

$$\Lambda = \begin{bmatrix} 1 & \rho & \rho^2 & . & . & \rho^{M-2} & \rho^{M-1} \\ \rho & 1 & \rho & . & . & . & \rho^{M-3} & \rho^{M-2} \\ \rho^2 & \rho & 1 & . & . & . & \rho^{M-4} & \rho^{M-3} \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ \rho^{M-2} & \rho^{M-3} & \rho^{M-4} & . & . & . & 1 & \rho \\ \rho^{M-1} & \rho^{M-2} & \rho^{M-3} & . & . & . & \rho & 1 \end{bmatrix} \tag{18}$$

Eqs.(11, 17 &18) are the basic formulas of our analysis in this manuscript.

The PDF of the output of the i<u>th</u> test tap is given by the Laplace inverse of Eq.(11), in the case of χ^2 distribution with two degrees of freedom, or Eq.(17), in the case of χ^2 fluctuation with four degrees of freedom, after some minor modifications have been made. If the i<u>th</u> test tap contains noise alone,Ω=0 is set that is the average noise power at the receiver input ψ. If the i<u>th</u> range cell contains a return from the primary target, it rests without any modifications, where Ω represents the strength of the target return at the receiver input. On the other hand, if the i<u>th</u> test cell is corrupted by interfering target return, Ω must be replaced by I, where I denotes the interference-to-noise (INR) at the receiver input.

III. Performance of CFAR Detectors in Benign Environments

A benign environment in the present context is defined as an environment of additive Gaussian noise and possibly additive broadband jamming that can also be modeled as Gaussian. The statistical model with uniform clutter background describes the classical signal situation with stationary noise in the reference window. In this model, two signal cases are of interest: (a) one target in the test cell in front of an otherwise uniform background, (b) uniform noise situation throughout the reference window. In both cases, the noise neighborhood has a uniform statistic, i.e., the random variables Q_1, Q_2, ……., Q_N in the reference window are assumed to be statistically independent and identically distributed. In the absence of the target case, the random variable ν of the cell under test is assumed to be statistically independent of the neighborhood and subject to the same distribution as the random variables Q_i's.

The essence of CFAR is to compare the decision statistic ν with an adaptive threshold TZ, see Fig. (1). the threshold coefficient T is a constant scale factor used to achieve a desired false alarm rate for a given window size N when the background noise is homogeneous. The statistic Z is a random variable whose distribution depends upon the particular CFAR scheme chosen and the underlying distribution of each of the reference range samples. Since the unknown noise power level estimate Z is a random variable, the processor performance is determined by calculating the average values of false alarm and detection probabilities. The false alarm probability P_{fa} is defined as

$$P_{fa} \triangleq \int_0^\infty P_{fa}(thr = TZ)\ p_Z(Z)\ dZ \tag{19a}$$

with

$$P_{fa}(thr = \alpha) \triangleq \int_\alpha^\infty p_\nu(x|\Omega = 0)\ dx \tag{19b}$$

In the above expression, **thr** denotes the decision threshold and $p_Z(.)$ represents the PDF of the noise power level estimate Z. In terms of the CF of the random variable Z, its PDF can be obtained by taking the inverse Laplace transformation of this CF. Thus,

$$\begin{aligned} p_Z(y) &= \frac{1}{2\pi i} \oint_c C_Z(\omega)\ \exp(\omega y)\ d\omega \\ &= \sum_j res\left\{C_Z(\omega)\exp(\omega y), \omega_j\right\} \end{aligned} \tag{20}$$

The contour of integration 'c' lies in the right of all singularities of $C_Z(\omega)$ beginning in the left half-plane, ω_j's are the poles of $C_Z(\omega)$, and res(.) denotes the residue. Hence,

$$\int_\alpha^\infty p_\nu(y)dy = -\sum_k res\left\{\frac{C_\nu(\omega|\Omega = 0)e^{\alpha\omega}}{\omega}, \omega_k\right\} \tag{21}$$

Finally, the substitution of Eqs.(21, 20 & 19) into the definition of P_{fa} results in

$$P_{fa} = -\sum_\ell res\left\{\frac{C_\nu(\omega|\Omega = 0)\,C_Z(-T\omega)}{\omega}, \omega_\ell\right\} \tag{22}$$

Similarly, the detection probability can be easily calculated as

$$P_d = -\sum_j res\left\{\frac{C_\nu(\omega|\Omega \neq 0)\,C_Z(-T\omega)}{\omega}, \omega_j\right\} \tag{23}$$

The detection probability of a CFAR processor for χ^2 targets with two degrees of freedom can be obtained by substituting Eq.(11) into the definition of P_d, which gives

$$\begin{aligned} P_d &= -\sum_j res\left\{\prod_{i=1}^{M} \frac{d_i}{\omega + d_i} \frac{C_Z(-T\omega)}{\omega}, \omega_j\right\} \\ &= \sum_{j=1}^{M} \prod_{\substack{i=1 \\ i \neq j}}^{M} \frac{d_i}{d_i - d_j}\ C_Z(T\,d_j) \end{aligned} \tag{24}$$

On the other hand, when the fluctuation of the primary target follows the χ^2 distribution with four degrees of freedom, $C_\nu(\omega)$ takes the following simplified form

$$\begin{aligned} C_\nu(\omega) &= \prod_{j=1}^{M} b_j^{\,2} \frac{\omega + 1}{(\omega + b_j)^2} \\ &= \sum_{j=1}^{M} \left\{\frac{g_j}{\omega + b_j} + \frac{h_j}{(\omega + b_j)^2}\right\} \end{aligned} \tag{25}$$

where

$$h_j \triangleq b_j^{\,2}\left(1-b_j\right)\prod_{\substack{k=1\\k\neq j}}^{M} b_k^{\,2}\,\frac{1-b_j}{\left(b_k-b_j\right)^2} \tag{26}$$

and

$$g_j \triangleq h_j\left\{\frac{M}{1-b_j}-2\sum_{\substack{\ell=1\\ \ell\neq j}}^{M}\frac{1}{b_\ell-b_j}\right\} \tag{27}$$

The substitution of Eq.(25) into the definition of the detection probability gives the processor detection performance for χ^2 targets with four degrees of freedom, which becomes

$$P_d=\sum_{j=1}^{M}\left\{\frac{g_j}{b_j}C_Z\left(Tb_j\right)+\frac{h_j}{b_j}\left[\frac{C_Z\left(Tb_j\right)}{b_j}+\frac{d}{d\omega}\left\{C_Z\left(-T\omega\right)\right\}\Big|_{\omega=-b_j}\right]\right\} \tag{28}$$

To obtain the relationship between the false alarm probability P_{fa} and the thresholding constant T, it is assumed that no target is present in the test cell (A=0). Therefore, as M-pulse non-coherent integration is used, P_{fa} can be calculated as

$$P_{fa}=-\frac{1}{\Gamma(M)}\frac{d^{M-1}}{d\omega^{M-1}}\left\{\frac{C_Z\left(-T\omega\right)}{\omega}\right\}\Bigg|_{\omega=-1} \tag{29}$$

In this case, $C_v(\omega)$ has an Mth order pole at ω=-1. Thus,

$$P_{fa}=\sum_{j=0}^{M-1}\frac{1}{\Gamma(j+1)}\frac{d^j}{d\omega^j}\left\{C_Z(-T\omega)\right\}\Big|_{\omega=-1} \tag{30}$$

It is of important to note that the above expression for the rate of false alarm is verified either for χ^2 targets with two degrees of freedom or for χ^2 targets with four degrees of freedom. Moreover, the characteristic function of the noise power level estimate 'Z' is the backbone of the processor performance analysis, as shown in Eqs.(24. 28 & 30), either in homogeneous or non-homogeneous background environments. Therefore, our scope in the following subsections is to calculate this important quantity for the two processors under consideration.

a) Cell-Averaging (CA) Detector

A simple approach to achieve the CFAR condition is to set the detection threshold on the basis of the average noise power in a given number of reference cells where each of these cells is assumed to contain no targets. Such a scheme is denoted as cell-averaging (CA) CFAR processor. This detector is specifically tailored to provide good estimates of the noise power in the exponential PDF. In this CFAR detection technique, the total noise power level is estimated by the sum of N range cells of the reference window. This is a complete, sufficient statistic for the noise power ψ under the assumption of exponentially distributed homogeneous background. In other words, the noise power is estimated as

$$Z_{CA}\triangleq\sum_{i=1}^{N}\sum_{j=1}^{M}q_{ij}\triangleq\sum_{i=1}^{N}Q_i \quad \text{and} \quad Q_i\triangleq\sum_{j=1}^{M}q_{ij} \tag{31}$$

Under the assumption that the surrounding range cells contain independent Gaussian noise samples with the same variance, Z_{CA} is the maximum likelihood estimate of the common variance. In the absence of the radar target, each cell of the reference window has a CF of the same form as that of ν with the exception that Ω must be set to be zero. Therefore, the elimination of Ω in either Eq.(11) or Eq.(17) leads to

$$C_Q(\omega)=\left(\frac{1}{\omega+1}\right)^M \tag{32}$$

Since the random variables Qi's are assumed to be statistically independent, ZCA has a CF given by

$$C_{Z_{CA}}(\omega)=\left(\frac{1}{\omega+1}\right)^L \;\;\&\;\; L\triangleq MN \tag{33}$$

Once the CF of the noise power level estimate is calculated, the detector performance evaluation can be easily carried out when the primary target fluctuates in accordance with $\chi 2$ model either with two or four degrees of freedom as shown in Eqs.(24, 28 & 30), where

$$\frac{d^k}{d\omega^k}\left\{C_{Z_{CA}}(\alpha\omega)\right\}=(-\alpha)^K\,(L)_K\left(\frac{1}{\alpha\omega+1}\right)^{L+K}$$

(34)The Pochammer symbol $(x)\lambda$ is as defined in [[El_Mashade, 1994]]:

$$(x)_\ell\triangleq\begin{cases}1 & for\quad \ell=0\\ x(x+1).....(x+\ell-1) & for\quad \ell=1,2,3,...\end{cases} \tag{35}$$

The rationale for the CA type of CFAR schemes is that by choosing the mean, the optimum CFAR processor in a homogeneous background when the reference cells contain IID observations governed by exponential distribution is achieved. As the size of the reference window increases, the detection probability approaches that of the optimum detector which is fixed threshold architecture [[El_Mashade, 2006].

b) Order-Statistics (OS) Detector

The performance of CA-CFAR detector degrades rapidly in non-ideal conditions caused by multiple targets and non-uniform clutter. The ordered-statistic (OS) CFAR is an alternative to the CA processor. The OS trades a small loss in detection performance, relative to the CA scheme, in ideal conditions for much less performance degradation in non-homogeneous background environments.

Order statistics characterize amplitude information by ranking observations in which differently ranked outputs can estimate different statistical properties of the distribution from which they stem. The order statistic corresponding to a rank K is found by taking the set of N observations $Q_1, Q_2, \ldots\ldots, Q_N$ and ordering them with respect to increasing magnitude in such a way that

$$Q_{(1)} \le \ldots \le Q_{(K-1)} \le Q_{(K)} \le Q_{(K+1)} \le \ldots \le Q_{(N)} \tag{36}$$

where $Q_{(K)}$ is the K<u>th</u> order statistic. The central idea of OS-CFAR procedure is to select one certain value from the above sequence and then used as an estimate Z for the average clutter power as observed in the reference window. Thus,

$$Z_{OS} = Q_{(K)} \;\&\; K \in \{1, 2, 3, \ldots\ldots\ldots\ldots, N\} \tag{37}$$

The OS scheme with parameter K will be denoted as OS(K). The value of K is generally chosen in such a way that the detection of radar target in homogeneous background environment is maximized.

In order to analyze the processor detection performance in uniform clutter background, the CF of the random variable Z_{OS} is required. This CF can be expressed as [[El_Mashade, 2011]]

$$C_{Z_{os}}(\omega) = \frac{K}{\Gamma(M)} \binom{N}{K} \sum_{i=0}^{K-1} \binom{K-1}{i} * (-1)^{K-i-1} \, \Phi_1(i,\omega) \tag{38}$$

where

$$\Phi_1(i,\omega) \triangleq \sum_{k_1=0}^{N-i-1} \sum_{k_2=0}^{N-i-1} \cdots\cdots\cdots \sum_{k_M=0}^{N-i-1} \frac{\Gamma(\beta)}{(\omega+N-i)^{\beta}} \left\{ \prod_{q=1}^{M} \left(\frac{1}{\Gamma(k_q+1)[\Gamma(q)]^{k_q}} \right) \Gamma(N-i) \right\} \tag{39}$$

with

$$\beta \triangleq M + \sum_{\alpha=2}^{M} (\alpha-1)\, k_\alpha \;\&\; \sum_{j=1}^{M} k_j = N - i - 1$$

(40)Once the CF of the noise power level estimate Z_{OS} is computed, the processor detection performance becomes an easy task, as we have previously proved. Finally, the ℓth derivative of this CF is given by:

$$\frac{d^\ell}{d\omega^\ell}\left\{C_{Z_{os}}(\omega)\right\} = \frac{K}{\Gamma(M)} \binom{N}{K} \sum_{i=0}^{K-1} \binom{K-1}{i} (-1)^{K-i-1} \frac{d^\ell}{d\omega^\ell}\left\{\Phi_1(i,\omega)\right\} \tag{41}$$

With

$$\frac{d^\ell}{d\omega^\ell}\left\{\Phi_1(i,\omega)\right\} = \sum_{k_1=0}^{N-i-1} \sum_{k_2=0}^{N-i-1} \cdots\cdots \sum_{k_M=0}^{N-i-1} \frac{(-1)^\ell (\beta)_\ell \, \Gamma(\beta)}{(\omega+N-i)^{\beta+\ell}} \left\{ \prod_{q=1}^{M} \left(\frac{1}{\Gamma(k_q+1)[\Gamma(q)]^{k_q}} \right) \Gamma(N-i) \right\} \tag{42}$$

A desirable CFAR scheme would of course be one that is insensitive to changes in the total noise power within the reference window cells so that a constant false alarm rate is maintained. This is actually the case of the two architectures under consideration.

c) Numerical Results

In this section, we present some representative numerical results, which will give an indication of the tightness of our previous analytical expressions. The performance of CFAR processors for partially-correlated χ^2 fluctuating targets is numerically evaluated for some parameter values and the results of these evaluations are presented in several sets of figures. The first set includes Figs.(2-3) and concerns with the detection performance of CA and OS schemes, respectively, for a number of integrated pulses of 2, 3, and 4 along with the single-sweep case which is included as a reference for comparison, when the radar target fluctuates in accordance with partially-correlated χ^2 model with two and four degrees of freedom. Since the performance of OS processor is strongly dependent on the ranking order parameter K, we choose the value that corresponds to the optimum detection performance in uniform noise background which is 21 for N=24 [8]. The displayed results of these figures show that for low values of SNR, the fully-correlated case (◎ ı =1.0) gives higher detection performance than the fully de-correlated (◎ ı =0) case. As the target return becomes stronger, an alternative version of the above behavior is observed, where the non-correlated performance surpasses the fully-correlated one. It is also noted that for M=2, the processor detection performance for ◎ 1 & ◎ ı =0 coincides with its detection performance for ◎ 2 & ◎ 1 =1.0. In addition, the non-correlated and fully-correlated detection performances embrace all the partially-correlated cases in either situation. This observation is common for any number of integrated pulses. Additionally, the partially-correlated χ^2 with ◎ 2 offers higher detection performance than that with ◎ 1 when the target return becomes stronger while the reverse of this behavior occurs when the target return is modest. Moreover, as M increases, the processor detection performance ameliorates and the reversing point is shifted towards lower values of SNR. By reversing point, we mean the point at which the detection performance changes its superiority from fully-correlated to fully-uncorrelated. All the presented results are obtained for a constant false alarm rate of 10^{-6} and a reference window of size 24 cells.

To make a comparison between the performances of the underlined CFAR schemes and that of the

optimum detector, the second group of illustrations includes Figs.(4-7). This category contains the partially-correlated χ^2 target homogeneous detection performance of the CA and OS schemes along with the optimum processor for M=2 and 4. The curves of this set are labeled in the CFAR procedure and the correlation coefficient 'ϱ_1', respectively. The indication OT, A, or O on a specified curve means that it is drawn for optimum, cell-averaging, or order-statistic detector, respectively. It is of importance to note that Figs.(4-5) describe the performance of the three processors when the primary target fluctuates following partially-correlated χ^2 model with two and four degrees of freedom, respectively, for M=2, while Figs.(6-7) depict the same thing for M=4. In any situation, it is noticed that the performance of CA algorithm is the much closer one to that of the optimum detector under any operating conditions and the OS scheme comes next. All the indicated remarks on the results of Figs.(2-3) are clearly demonstrated on the results of the present figures. For small SNR, the processor performance degrades as ϱ_1 decreases, while for large SNR, this performance improves as ϱ_1 decreases. This observation is noticed for partially-correlated χ^2 targets fluctuating with either two or four degrees of freedom. In addition, these plots illustrate the superiority of χ^2 fluctuation model with 4 degrees of freedom over that following χ^2 model with 2 degrees of freedom, especially for large SNR. As the number of non-coherently integrated pulses increases, the processor performance improves and less SNR value is needed to achieve the same level of detection.

In the next group of figures, we are concerned with what is known, in the world of radar target detection, as receiver operating characteristics (ROC's). These characteristics describe the detection probability as a function of the false alarm probability for a fixed target signal strength (SNR=const). This set of figures incorporates Figs.(8-9) which represent the ROC's of CA and OS(21), respectively. In these figures, the processor detection performance is plotted against the false alarm rate for a number of integrated pulses of 2 and 4 when the primary target fluctuates in accordance with χ^2 distribution with two and four degrees of freedom. Since the fully correlated and de-correlated cases enclose the partially-correlated situations, our evaluations are restricted to these two border limits (◎ $_1$=0 & ◎ $_1$=1). The reference window size is taken as 24 and the primary target signal strength as 5 dB. For comparison, the single sweep ROC's is also included in these plots. The curves of these figures are labeled in the number of post-detection integrated pulses and the correlation coefficient of the primary target returns (◎ $_1$). The displayed results show that for lower false alarm rates, the processor detection probability improves as ◎ $_1$ increases. As the false alarm rate increases, this behavior is rapidly reversed and the fully de-correlated detection performance surpasses that corresponding to the fully correlated case. Additionally, as the number of non-coherently integrated pulses increases, the reversing point or the critical rate, moves towards the lower false alarm rate and this is common as the target fluctuates following χ^2 with either two or four degrees of freedom. Moreover, the processor detection performance for χ^2 distribution with ◎ =2 is superior to that for χ^2 distribution with ◎ =1 in the case where the operating false alarm rate is higher than its critical value, otherwise, the processor detection performance when the fluctuation of the primary target follows χ^2 distribution with ◎ =1 surpasses its performance when this fluctuation obeys χ^2 distribution with ◎ =2.Keeping in mind that when M=2, the processor ROC's for ◎ =1 & ◎ $_1$=0 coincide with its ROC's for ◎ =2 & ◎ $_1$=1. The results of this group confirm the observed remarks on the previous groups as well as demonstrate our comments on the behavior of their figures. Additionally, the homogeneous detection performance of CA scheme is always superior to that of OS algorithm, under the same operating conditions, as predicted.

To illustrate the influence of the signal correlation on the processor detection performance, the last group is devoted to the processor detection performance against the correlation coefficient between the radar target returns given that the number of integrated pulses (M) as well as the signal strength (SNR) is held constant. Figs.(10-11) illustrate these characteristics for the underlined detectors, CA and OS(21), respectively, for SNR=-5, 0, and 5dB when M=1, 2, and 4, given that the radar target fluctuates in accordance with χ^2 distribution with ◎ =1Since the chosen signal strength is modest, the scheme detection performance improves as the target returns become highly correlated. This result is predicted since the correlation strengthens the weak signal returns in contrast to the case in which the signal returns are strengthened but the correlation weakens the signal returns. For the same reason, the rate of improvement decreases as the number of consecutive sweeps increases. Moreover, this rate of improvement increases as the target signal returns become weaker, given that the number of integrated pulses maintains constant. The two processors under

consideration give approximately the same level of detection for correlated signal returns without any superiority between them. This concluded remark is also predicted due to the fact that the CFAR schemes behave likely for the modest target signal strength, as demonstrated by the above mentioned results. Finally, if the primary target fluctuates in accordance with χ^2 distribution with ◎ =2,the underlined processors act the same behavior with minor improvement in each case.

IV. Performance of CFAR Schemes in Multitarget Environment

The performance of the CFAR algorithms for uniform clutter model is completely evaluated in the previous section. Clutter edges, on the other hand, are used to describe transition areas between regions with very different noise characteristics. Since it is concerned here with partially correlated χ^2 targets, this situation is of secondary scope. On the other hand, multiple target situations occur occasionally in radar signal processing when two or more targets are at a very similar range. The consequent masking of one target by the other is called suppression. These interferers can arise from either real object returns or pulsed noise jamming. From a statistical point of view, this implies that the reference samples, although still independent of one another, are no longer identically distributed. Let us now examine the dependence of the performance of the CFAR procedures on the accurate knowledge of the target fluctuation model when the reference window is contaminated with a fluctuating interfering target returns. In our study of the non-homogeneous background, the amplitudes of all the targets present amongst the candidates of the reference window are assumed to be of the same strength and to fluctuate in accordance with the partially-correlated χ^2 fluctuation model with correlation coefficient ϱ_i. The interference-to-noise ratio (INR) for each of the spurious targets taken as a common parameter is denoted by I.

a) Cell-Averaging (CA) Detector

To analyze the CA performance when the reference window no longer contains radar returns from a homogeneous background, the assumption of statistical independence of the reference cells is retained. Suppose that the reference window contains r cells from interfering target returns with background power of $\psi(1+I)$ and N-r cells from clear background with noise power ψ. Thus, the estimated total noise power level is obtained from

$$Z_{CA} = \sum_{i=1}^{r} q_i + \sum_{j=r+1}^{N} q_j \quad \triangleq \quad X + Y \tag{43}$$

The random variable representing the interfering target return 'X' has a CF of the same form as that given by Eq.(11), when the extraneous targets fluctuate in accordance with χ^2 model with two-degrees of freedom, and as that given by Eq.(16), in the case where the spurious targets are of the χ^2 model with four-degrees of freedom. In both cases, A (SNR of the primary target) should be replaced by I (INR of the secondary target). On the other hand, since each RV in Y represents the clear background return, it has a similar form, for its CF, as given by Eq.(32). Since the candidates of each type are assumed to be statistically independent, we have

$$C_{Z_{CA}}(\omega) = \left\{\prod_{j=1}^{M} \frac{f_j}{\omega + f_j}\right\}^r \left\{\frac{1}{\omega+1}\right\}^{M(N-r)} \quad \& \tag{44}$$

$$f_j \triangleq \frac{1}{1 + I\lambda_j} \quad for \quad \chi^2 \; with\, with \; 2-Degrees$$

and

$$C_{Z_{CA}}(\omega) = \left\{\prod_{j=1}^{M} e_k{}^2 \frac{\omega+1}{(\omega+e_k)^2}\right\}^r \left\{\frac{1}{\omega+1}\right\}^{M(N-r)} \quad \& \qquad e_k \triangleq \frac{1}{1+I\lambda_k/2} \quad for \quad \chi^2 \; with \; 4-Degrees \tag{45}$$

In the above expression, λ_i's represent the nonnegative eigenvalues of the correlation matrix, with $\varrho=\varrho_i$ in Eq. (18), associated with the spurious target returns. Finally, the derivative of the CF of the test statistic of the CA procedure, when the interfering targets fluctuate in accordance with χ^2 model of four degrees of freedom, can be easily computed as

$$\frac{d}{d\omega} C_{Z_{CA}}(\omega) = \left\{\prod_{j=1}^{M} e_k{}^2 \frac{\omega+1}{(\omega+e_k)^2}\right\}^r * \left\{\frac{1}{\omega+1}\right\}^{M(N-r)} \left\{\frac{M(2r-N)}{\omega+1} - 2\sum_{i=1}^{M} \frac{r}{\omega+e_i}\right\} \tag{46}$$

We repeat again that once CF of the noise power level, Z_{CA}, is obtained, the processor performance evaluation is completely determined, in the case where the fluctuation of the spurious targets follows the χ^2 model with either two or four degrees of freedom, as previously demonstrated.

b) Ordered-Statistics (OS) Detector

In order to analyze the processor performance when there are interfering target returns amongst the contents of the reference window, the assumption of

statistical independence of the reference cells is retained. Consider the situation where there are r reference samples contaminated by extraneous target returns, each with power level ψ(1+I), and the remaining N-r reference cells contain thermal noise only with power level ψ. Under these assumptions, the Kth ordered sample, which represents the noise power level estimate in the OS detector, has a cumulative distribution function (CDF) given by [El_Mashade, 2001]:

$$F_{Z_{OS}}(z) = \sum_{i=K}^{N} \sum_{j=\max(0,i-r)}^{\min(i,N-r)} \binom{N-r}{j}\binom{r}{i-j} \{F_c(z)\} \left[1-F_c(z)\right]^{N-r-j} \left[1-F_s(z)\right]^{r-i+j} \{F_s(z)\}^{i-j} \tag{47}$$

The CDF of the reference cell that contains a spurious target return, when this target fluctuates in accordance with χ^2 with two degrees of freedom, can be calculated as:

$$\begin{aligned} F_s(z) &= L^{-1}\left\{ \frac{1}{\omega} \prod_{\ell=1}^{M} \frac{1}{1+(1+I\lambda_\ell)\omega} \right\} \\ &= 1 - \sum_{\ell=1}^{M} \zeta_\ell \, e^{-c_\ell z} \end{aligned} \tag{48}$$

where L^{-1} denotes the Laplace inverse operator and

$$\zeta_\ell \triangleq \prod_{\substack{k=1\\k\neq\ell}}^{M} \frac{1+I\lambda_\ell}{I(\lambda_\ell-\lambda_k)} \quad \& \quad c_\ell \triangleq \frac{1}{1+I\lambda_\ell} \tag{49}$$

On the other hand, if the interfering target's fluctuation follows chi-square model with four degrees of freedom, $F_s(z)$ takes the form

$$\begin{aligned} F_s(z) &= L^{-1}\left\{ \frac{1}{\omega} \prod_{j=1}^{M} \varepsilon_j^{\,2} \frac{\omega+1}{(\omega+\varepsilon_j)^2} \right\} \\ &= 1 - \sum_{j=1}^{M} \left(a_j + z t_j\right) \exp(-\varepsilon_j z) \end{aligned} \tag{50}$$

with

$$a_j \triangleq \varepsilon_j (1-\varepsilon_j)^M \prod_{\substack{i=1\\i\neq j}}^{M} \left(\frac{\varepsilon_i}{\varepsilon_i-\varepsilon_j}\right)^2 * \left\{ \frac{M}{1-\varepsilon_j} + \frac{1}{\varepsilon_j} - \sum_{\substack{\ell=1\\\ell\neq j}}^{M} \frac{2}{\varepsilon_\ell-\varepsilon_j} \right\} \tag{51}$$

and

$$t_j \triangleq \varepsilon_j (1-\varepsilon_j) \prod_{\substack{k=1\\k\neq j}}^{M} \varepsilon_k^{\,2} \frac{1-\varepsilon_j}{(\varepsilon_k-\varepsilon_j)^2} \quad \& \quad \varepsilon_j \triangleq \left(1 + I\frac{\lambda_j}{2}\right)^{-1} \tag{52}$$

Since F(x)=1-[1-F(x)], Eq.(47) can be written in another simpler form as [15]:

$$F_{Z_{OS}}(z) = \sum_{i=K}^{N} \sum_{j=\max(0,i-r)}^{\min(i,N-r)} \binom{N-r}{j}\binom{r}{i-j} * \sum_{k=0}^{j}\binom{j}{k}(-1)^{j-k}\left[1-F_c(z)\right]^{N-r-k} * \sum_{\ell=0}^{i-j}\binom{i-j}{\ell}(-1)^{i-j-\ell}\left[1-F_s(z)\right]^{r-\ell} \tag{53}$$

By taking the Laplace inverse of Eq.(32) and then substitued, along with Eq.(48), into Eq.(53) one obtains, for χ^2 target fluctuation with two degrees of freedom,

$$F_{Z_{OS}}(z) = \sum_{i=K}^{N} \sum_{j=\max(0,i-r)}^{\min(i,N-r)} \binom{N-r}{j}\binom{r}{i-j} * \sum_{k=0}^{j}\sum_{\ell=0}^{i-j}\binom{j}{k}\binom{i-j}{\ell}(-1)^{i-k-\ell}\left\{\sum_{m=0}^{M-1}\frac{z^m}{\Gamma(m+1)}\exp(-z)\right\}^{N-r-k} \left\{\sum_{n=1}^{M}\zeta_n \exp(-c_n z)\right\}^{r-\ell} \tag{54}$$

On the other hand, if the extraneous targets fluctuate according to χ^2 model with four degrees of freedom, Eq.(53) takes the form

$$F_{Z_{OS}}(z) = \sum_{i=K}^{N} \sum_{j=\max(0,i-r)}^{\min(i,N-r)} \binom{N-r}{j}\binom{r}{i-j} * \sum_{k=0}^{j}\sum_{\ell=0}^{i-j}\binom{j}{k}\binom{i-j}{\ell}(-1)^{i-k-\ell}\left\{\sum_{m=0}^{M-1}\frac{z^m}{\Gamma(m+1)}\exp(-z)\right\}^{N-r-k} \left\{\sum_{n=1}^{M}(a_n + z t_n)\exp(-\varepsilon_n z)\right\}^{r-\ell} \tag{55}$$

To determine the detection performance of the OS-CFAR processor, it is important to calculate the Laplace transform for its test statistic, where the false alarm and detection probabilities are completely dependent on this transformation along with its derivatives with respect to ω. The ω-domain representation of Eq.(54) is:

$$\Phi_{F_\kappa}(\omega) = \sum_{i=K}^{N} \sum_{j=\max(0,i-r)}^{\min(i,N-r)} \binom{N-r}{j}\binom{r}{i-j} \sum_{k=0}^{j}\sum_{\ell=0}^{i-j}\binom{j}{k}\binom{i-j}{\ell} (-1)^{i-k-\ell} \sum_{\theta_0=0}^{N-r-k}\sum_{\theta_1=0}^{N-r-k}\cdots\sum_{\theta_{M-1}=0}^{N-r-k} \frac{\Psi(N-r-k;\theta_0,\ldots,\theta_{M-1})}{\prod_{\nu=0}^{M-1}[\Gamma(\nu+1)]^{\theta_\nu}} * \sum_{\vartheta_1=0}^{r-\ell}\sum_{\vartheta_2=0}^{r-\ell}\cdots\cdots\sum_{\vartheta_M=0}^{r-\ell}\Psi(r-\ell;\vartheta_1,\ldots,\vartheta_M)\prod_{\xi=1}^{M}(\zeta_\xi)^{\vartheta_\xi} * \frac{\Gamma\left(\sum_{\gamma=0}^{M-1}\gamma\theta_\gamma+1\right)}{\left(\omega+N-r-k+\sum_{\eta=1}^{M}\vartheta_\eta c_\eta\right)^{\sum_{\gamma=0}^{M-1}\gamma\theta_\gamma+1}} \tag{56}$$

where the definition of the term $\Psi(S; i_1, i_2, \ldots\ldots, i_M)$ is as follows:

$$\Psi(S;i_1,i_2,\ldots,i_M) \triangleq \begin{cases} \dfrac{S!}{\prod_{\ell=1}^{M} i_\ell!} & \text{for} \quad S=\sum_{\ell=1}^{M} i_\ell \\ 0 & \text{for} \quad S\neq\sum_{\ell=1}^{M} i_\ell \end{cases} \tag{57}$$

On the other hand, the Laplace transformation of

Eq.(55) is analytically very complicated and therefore the numerical techniques is the only way that allows us to compute this transformation.

Again, the OS-CFAR processor performance is highly dependent upon the value of K. For example, if a single extraneous target appears in the reference window of appreciable magnitude, it occupies the highest ranked cell with high probability. If K is chosen to be N, the estimate will almost always set the threshold based on the value of interfering target. This increases the overall threshold and may lead to a target miss. If, on the other hand, K is chosen to be less than the maximum value, the OS-CFAR scheme will be influenced only slightly for up to N-K spurious targets.

c) Numerical Results

Here, we provide a variety of numerical results for the performance of CA- and OS-CFAR processors in multiple target environments. Since the optimum value of K, for a reference window of size 24, is 21, it has been assumed that there are three interfering target returns amongst the contents of the estimation cells. This value of extraneous target returns is the maximum allowable value before the OS performance degradation occurs. Our numerical results are obtained for a possible practical situation where the primary and the secondary interfering targets fluctuate in accordance with the χ^2 fluctuation model with the same correlation coefficient ($\varrho_1=\varrho_2=$◎), and of equal target return strength (INR=SNR). Additionally, for our new results to be comparable with the older ones (in the absence of outlying targets), the design rate of false alarm is held unchanged ($P_{fa}=10^{-6}$). The obtained results are classified into categories. The first category, which includes Figs.(12,13), depicts the detection performance of the CA and OS(21) schemes, respectively, under the same operating conditions as in Figs.(2,3) with the exception that three spurious targets are allowed to be present amongst the candidates of the reference set. The curves of these figures are labeled in the number of integrated pulses (M), the strength of correlation between the target returns (◎)and the parameter that represents the degrees of freedom (◎). For comparison, the single sweep case is included in these plots. In addition to the partially-correlated cases, the well-known four Swerling's models are also included. It is important to note that the full scale of the P_d axis in this case of CA scheme has a maximum value of 75%. There is another interesting point about the reaction of CFAR schemes against χ^2 fluctuation model where the SWII & SWIII performances are coincide for M=2. The displayed results show that the behavior of the CFAR processors in the presence of extraneous targets is similar to that in their absence. By comparing the results of Fig.(12) to the corresponding ones of Fig.(13), it is observed that intolerable masking of the primary target occurs in the case of CA procedure and the OS(21) architecture is capable to resolve multiple targets in the reference window as long as their number is within its allowable values ($r \leq N-K$). All the concluded remarks about the ideal performance of CA and OS(21) schemes are also observed on their multi-target performance given that the target under test along with the interfering ones fluctuate obeying χ^2 model with 2 and 4 degrees of freedom. For the comparison between the behaviors of the considered detectors, against the presence of outlying targets to be clarified, the next category combines their performances in the same figure. This category, containing Figs.(14-15), illustrates the multiple-target detection performance of CA and OS(21) processors when the principal and the spurious targets fluctuate following χ^2 model with 2 and 4 degrees of freedom, respectively. As a reference level of performance, the detection performance of the optimum scheme is also included in these figures. The superiority of the OS performance is very clear under any level of correlation and the χ^2 fluctuation model with 4 degrees of freedom gives always an improved performance than that fluctuating with 2 degrees of freedom. This result is predicted since all the reference cells participate in constructing the detection threshold in the case of CA processor while the K^{th} sample is the only one that is responsible for building the detection decision. This means that the presence of interferers will raise the threshold in the case of CA scheme while it has no effect on the detection threshold in the OS algorithm, given that their number is within its allowable values. Additionally, the displayed results confirm the improvement of the processor detection performance with increasing the number of post-detection integrated pulses, as the theoretical analysis demonstrates. The next set of curves (Figs.16-17) illustrates the required SNR to achieve an operating point ($P_{fa}=10^{-6}$ و $P_d=90\%$) for the two CFAR processors under investigation, as a function of the consecutive pulses correlation coefficient ($\varrho_1=\varrho_2=\varrho$), for several values of post-detection integrated pulses when the fluctuation model of the tested target as well as the secondary interfering ones obeys χ^2 distribution with 2 and 4 degrees of freedom, respectively. The candidates of this set are labeled in the CFAR processor, number of integrated pulses, and the number of reference cells that are contaminated with extraneous target returns. It is of importance to note

that r=0 means homogeneous background which is free from any interferers. In that case, the CA processor needs less strengthened target signal compared to the OS scheme, to attain the required level of detection for a given rate of false alarm. In addition, the required SNR increases as ϱ increases, given that M along with r are held fixed. In multi-target situation, on the other hand, the CA procedure is unable to reach the given level of detection (90%) for the specified values of its parameters. For this reason, Fig.(16) shows the multiple-target detection performance of the OS processor only since the CA scheme is unable to achieve a detection probability of 0.9 value in the presence of 3 outlying targets, for either M=2 or M=4. Since the required value of the probability of detection is high, the processor detection performance for χ^2 fluctuating targets with 4 degrees of freedom is better than its performance for those fluctuating in accordance with χ^2 model with 2 degrees of freedom. This is the fact that Fig.(17) demonstrates. Finally, the variations of the false alarm rate with the strength of the interfering target returns are plotted, in the last category of curves, for the underlined architectures when they are reacted to χ^2 fluctuating targets with 2 and 4 degrees of freedom. This set of figures includes Figs.(18-19). In this case, the primary and the secondary outlying targets are assumed to be fluctuating with the same degree of freedom and our numerical results are constrained to the two limits of the correlation coefficient (ϱ=0, ϱ=1) which are corresponding to the well-known Swerling's models. The curves of these figures are labeled in the specified processor (A: for cell-Averaging, O: for Order-statistic), number of integrated pulses M and the Swerling fluctuation model. The numerical results of these figures are given for a design rate of false alarm of 10^{-6} and a reference window of size 24 samples. The interfering targets are assumed to exhibit χ^2 statistics with full correlation (ϱ_i=1), which corresponds to SWI, when the degrees of freedom is 2, or SWIII, when the degrees of freedom is 4. On the other hand, when the outlying targets exhibit χ^2 statistics with null correlation (ϱ_i=0), the resulting fluctuation model is SWII, in the case where the degrees of freedom is 2, or SWIV, in the case where the degrees of freedom is 4. The results of these figures show that the false alarm rate performance of the CA processor degrades as the strength of the interfering target return (INR) increases, and the rate of degradation decreases as ϱ increases or the degree of freedom decreases. This statement is common for the CFAR processors considered here. However, the OS(21) scheme has the lowest rate of degradation and consequently, it is the only processor capable to maintain the false alarm rate at approximately the desired value, given that the number of spurious target returns doesn't exceed its allowable values. In addition, when the INR becomes very high, the behavior of the OS detector is independent on the correlation coefficient ϱ and tends to be constant. Moreover, the OS false alarm rate performance improves as the number of post-detection integrated pulses increases, which is excluded from offer in the CA technique. This result is expected since the largest interfering target returns occupy the top ranked cells and therefore they are not incorporated in the estimation of the background noise power level. In other words, the noise estimate is free of extraneous target returns and therefore it represents the homogeneous background environment. As a result, the false alarm rate performance of the OS procedure improves as the number of non-coherently integrated pulses increases.

V. Conclusions

In this paper, the detection probability of a radar receiver that post-detection integrates M pulses of an exponentially correlated signal from a Rayleigh target in thermal noise is calculated. At the limiting correlation coefficients, ρ=1 and ρ=0, the analysis yields, respectively, the well-known Swerling cases 1 & 2. In addition, the probability of detection of the sum of M square-law detected pulses is computed for the case where the signal fluctuation obeys χ^2 statistics with four degrees of freedom. Swerling's well-known cases 3 & 4 represent the situations where the signal is completely correlated and de-correlated, respectively, from pulse to pulse. Moreover, the performance of CFAR processors for partial signal correlation in these two situations has been analyzed. These processors include CA detectorwith the best homogeneous performance, and OS procedure, which trades a small loss in homogeneous performance, relative to CA, for much less performance degradation in target multiplicity environments. The exact false alarm and detection probabilities have been derived, in the absence as well as in the presence of extraneous targets, for the condition of partial signal correlation. The primary and the secondary interfering targets are assumed to be fluctuating in accordance with χ^2-distribution with two and four degrees of freedom. The analytical results have been used to develop a complete set of performance curves including ROC's, detection probability in homogeneous and multi-target

situations, required SNR to achieve a prescribed values of P_{fa} & P_d, and the variation of false alarm rate with the strength of interfering target returns that may exist amongst the contents of the estimation set. As expected, the detection performance of the CFAR detectors for partially-correlated χ^2 targets with two degrees of freedom is between that for SWI targets and that for SWII models. On the other hand, the detection performance of the CFAR schemes for partially correlated χ^2 targets with four degrees of freedom lies between that for SWIII fluctuation model and that for SWIV targets. In any one of these fluctuating families, more per pulse signal-to-noise ratio is required to achieve a prescribed probability of detection as the signal correlation increases from zero to unity. In addition, the false alarm rate increases with the signal correlation and the OS architecture is the only, relative to the CA scheme, processor that is capable to maintain a constant rate of false alarm, irrespective to the interference level, in the case where the spurious target returns occupy the top ranked cells and they are within their allowable values. As a final conclusion, the detection performance of a CFAR procedure is related to the target model, the number of post-detection integrated pulses, and the average power of the target. When the target signal fluctuates obeying χ^2 statistics, the signal components are correlated from pulse to pulse and this correlation degrades the processor performance. A common and accepted practice in radar system design to mitigate the effect of target fluctuation is to provide frequency diversity to de-correlate the signal from pulse to pulse. While this technique is effective, it requires additional system complexity and cost.

The results for partial-correlation fall between those for the two extremes of complete de-correlation and complete correlation. Thus, to estimate the performance for partially-correlated pulses, interpolation between the results for completely correlated and de-correlated conditions can be used as an approximation. Extension to the partially-correlated $\chi 2$ target with 2κ degrees of freedom, where $\kappa>2$, is relatively straightforward.

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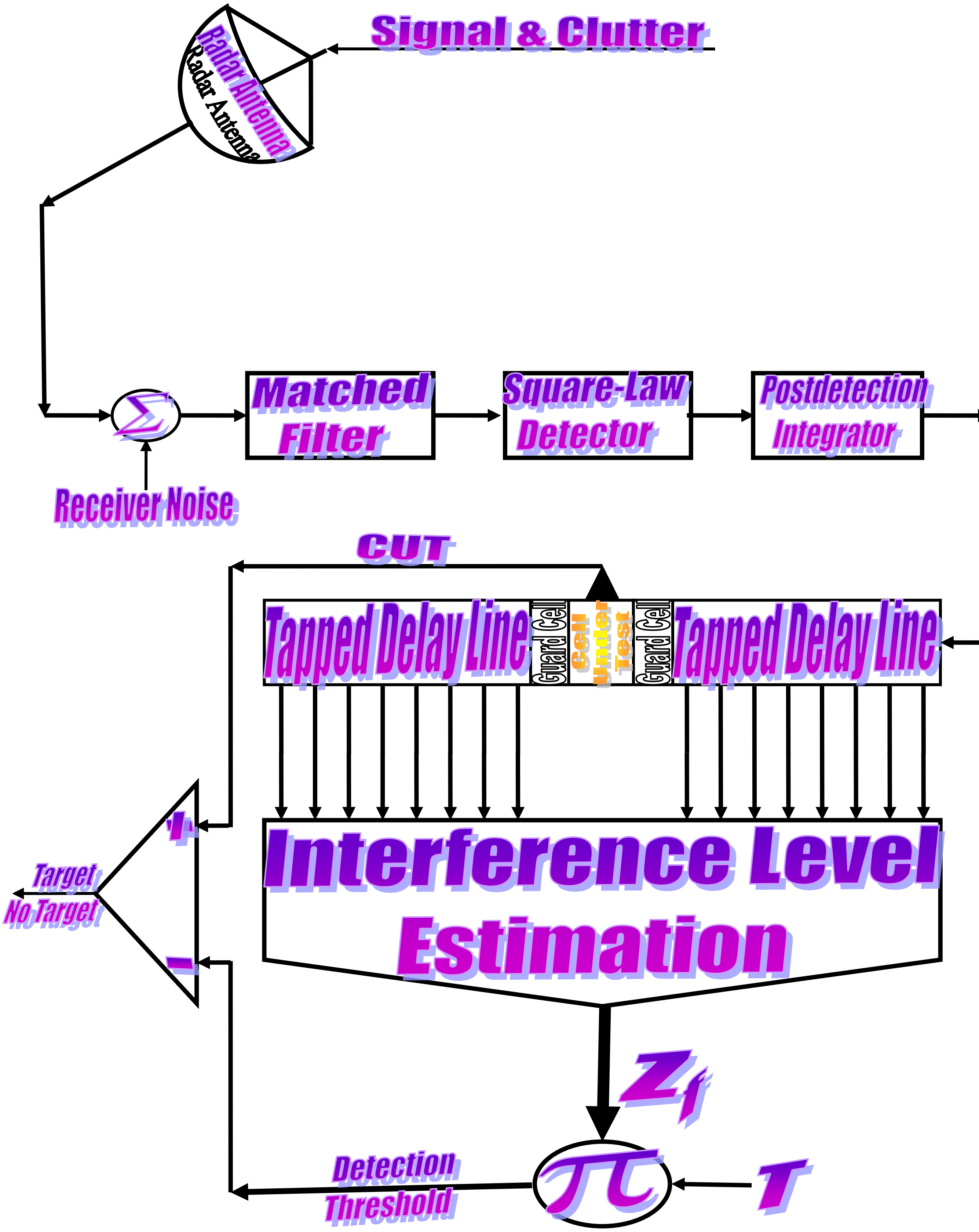

Fig.(1) Architecture of adaptive processor with postdetection integration

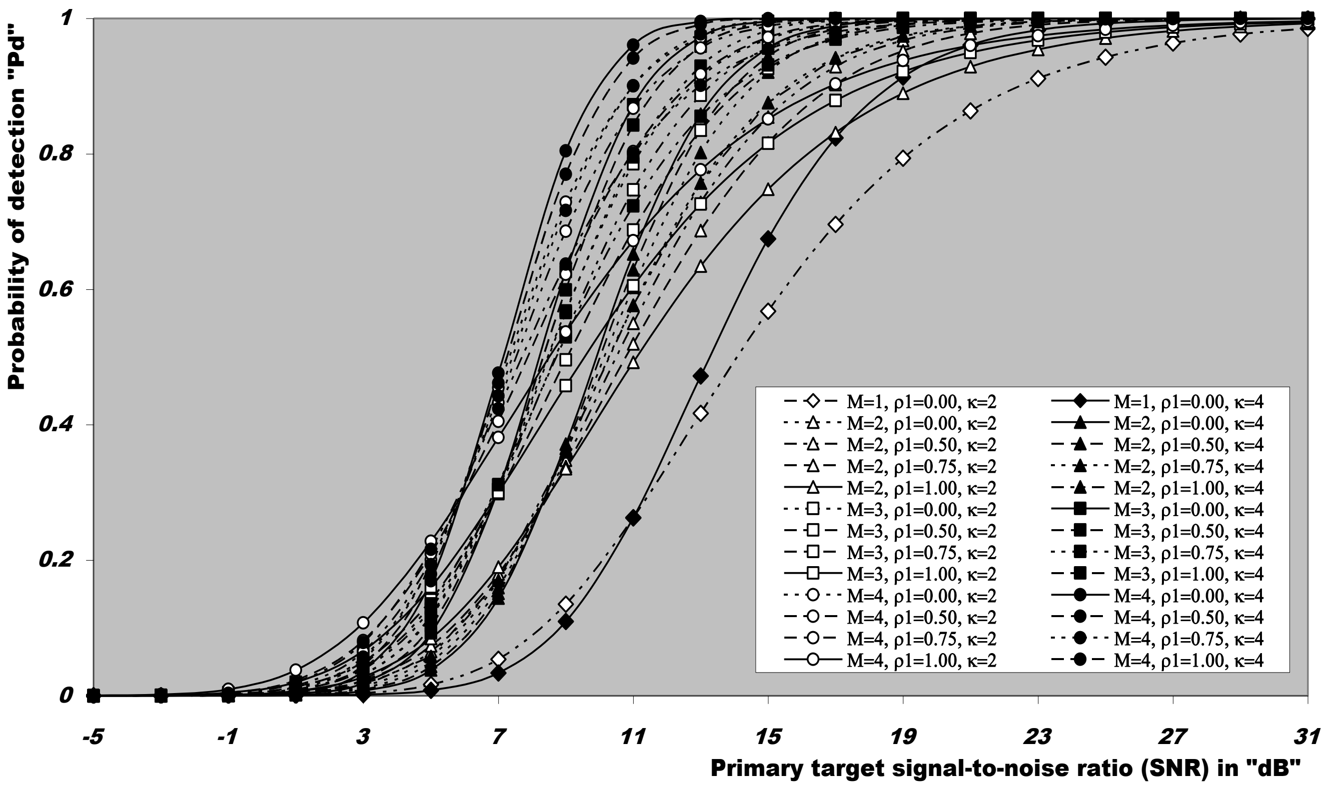

Fig.(2) M-sweeps ideal detection performance of CA scheme for partially-correlated $\chi 2$ targets with 2 & 4 degrees of freedom when N=24, and Pfa=1.0E-6.

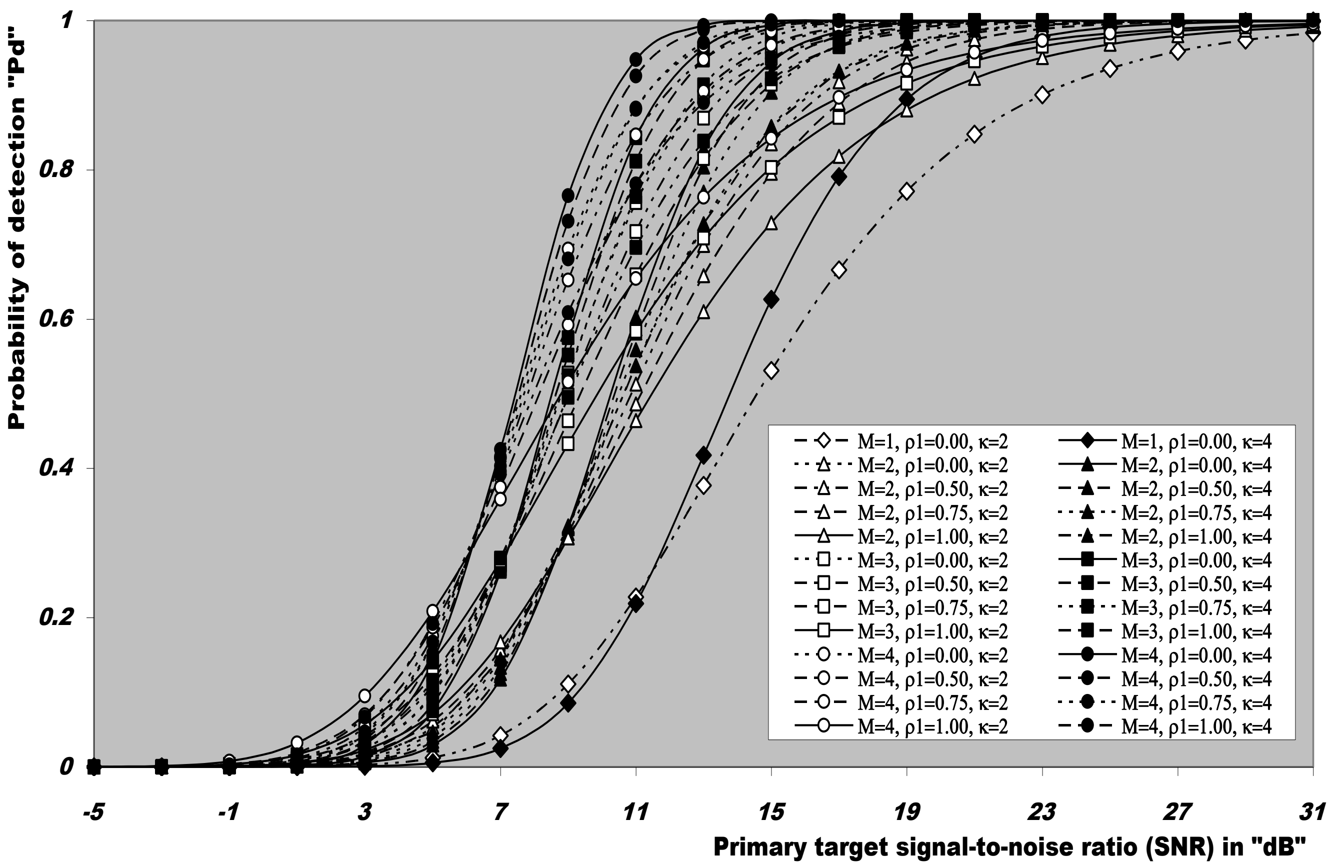

Fig.(3) M-sweeps ideal detection performance of OS(21) scheme for partially-correlated $\chi 2$ targets with 2 & 4 degrees of freedom when N=24, and Pfa=1.0E-6.

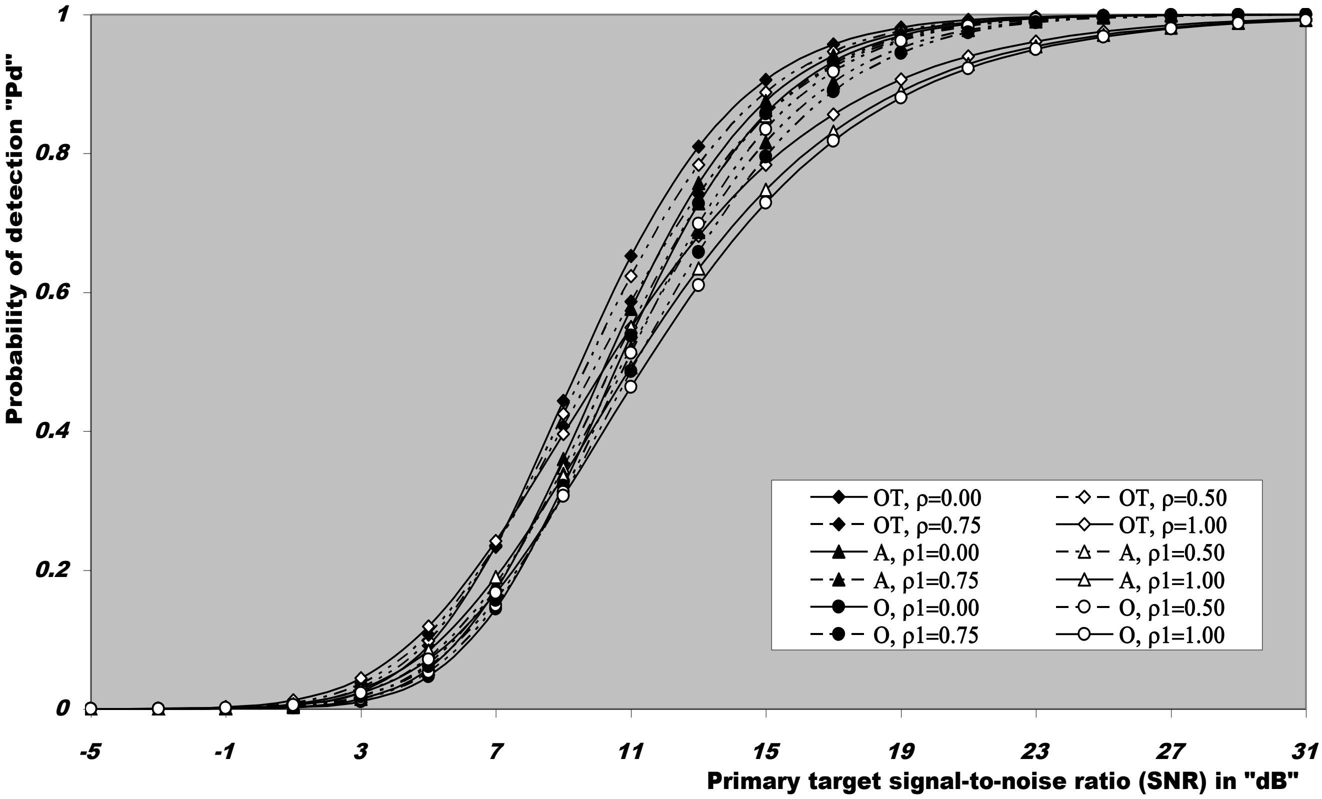

Fig.(4) M-sweeps ideal detction performance of CFAR processors for partially-correlated $\chi 2$ targets with two degrees of freedom when N=24, M=2, Pfa=1.0E-6.

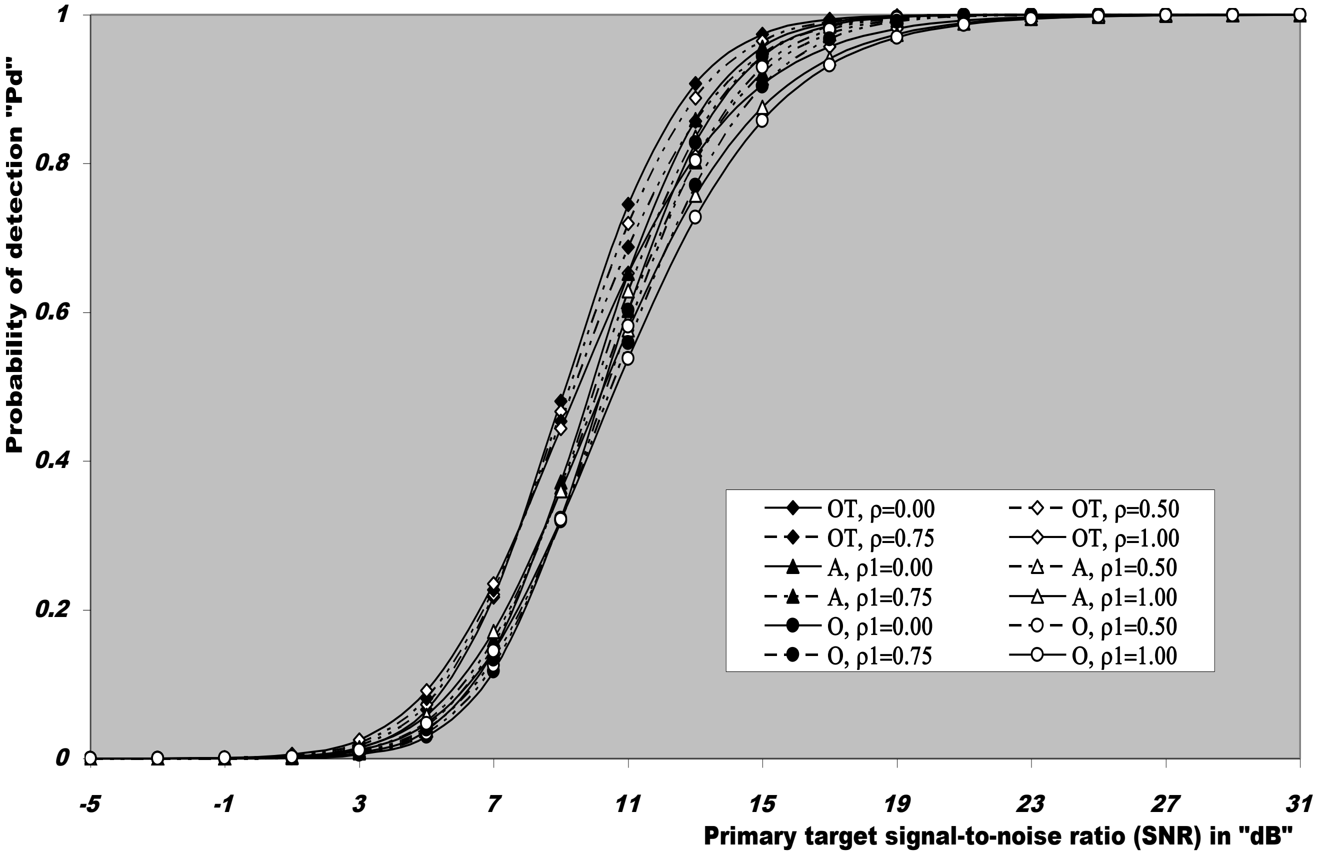

Fig.(5) M-sweeps ideal detction performance of CFAR processors for partially-correlated $\chi 2$ targets with four degrees of freedom when N=24, M=2, Pfa=1.0E-6.

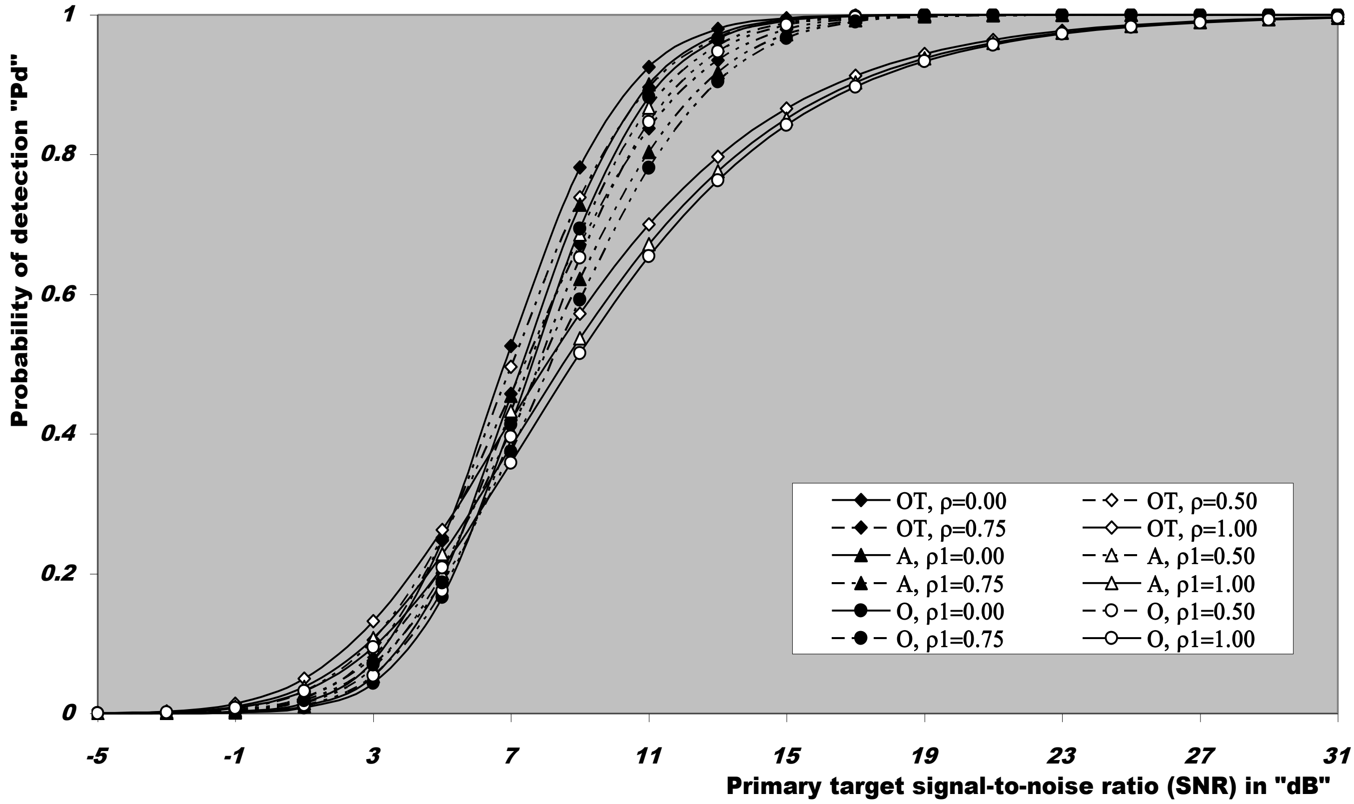

Fig.(6) M-sweeps ideal detction performance of CFAR processors for partially-correlated $\chi 2$ targets with two degrees of freedom when N=24, M=4, Pfa=1.0E-6.

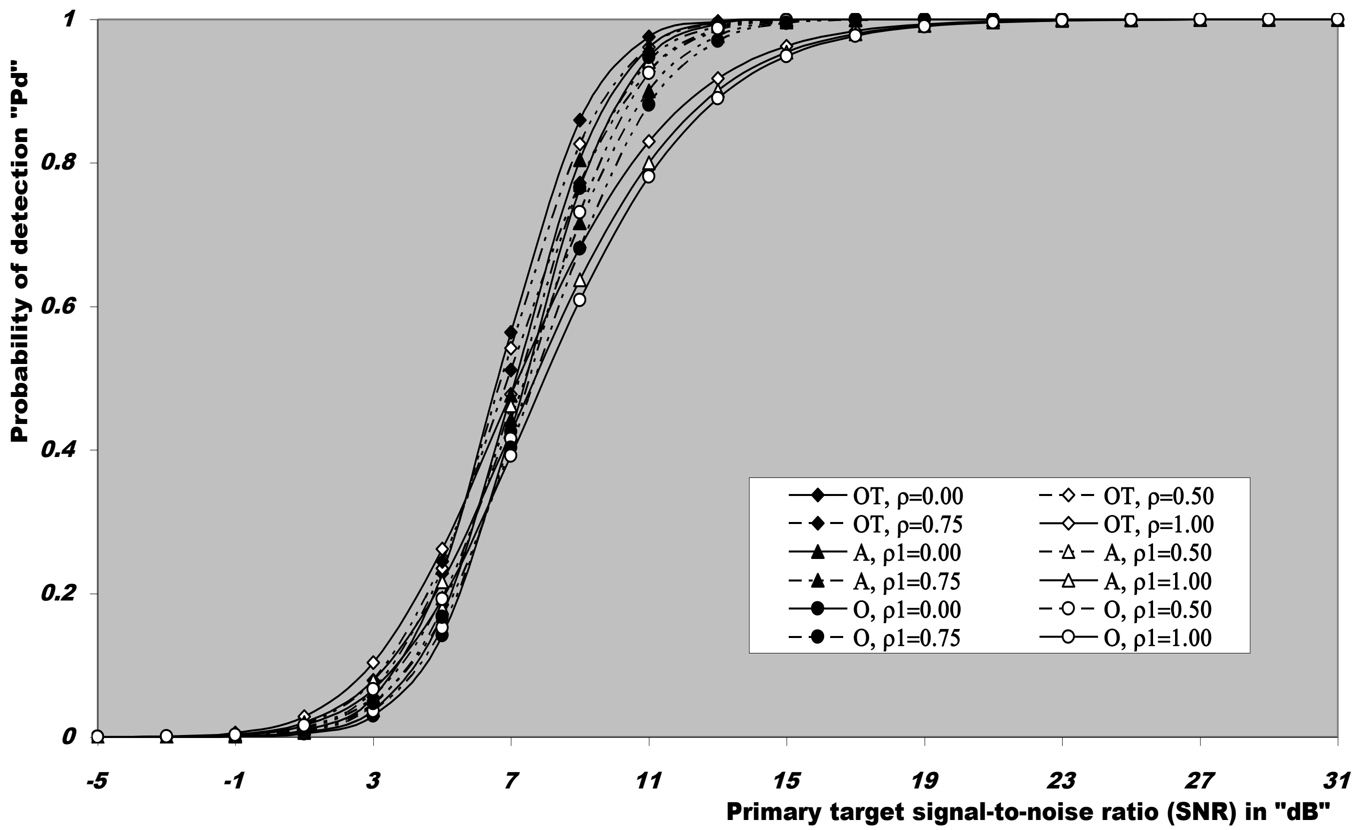

Fig.(7) M-sweeps ideal detction performance of CFAR processors for partially-correlated $\chi 2$ targets with four degrees of freedom when N=24, M=4, Pfa=1.0E-6.

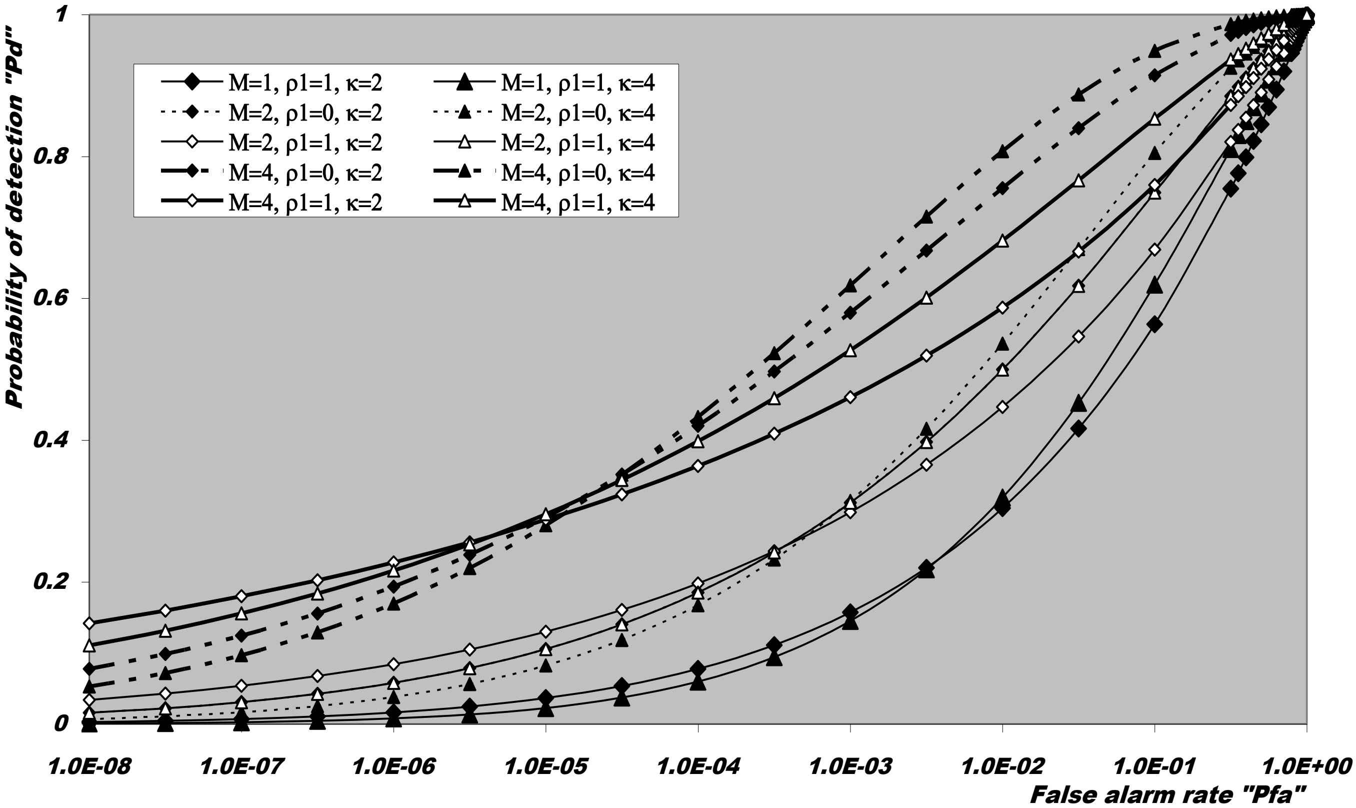

Fig.(8) M-sweeps receiver operating characteristics (ROC's) of CA processor for partially-correlated $\chi 2$ targets in homogeneous situation when N=24 and Ω=5dB.

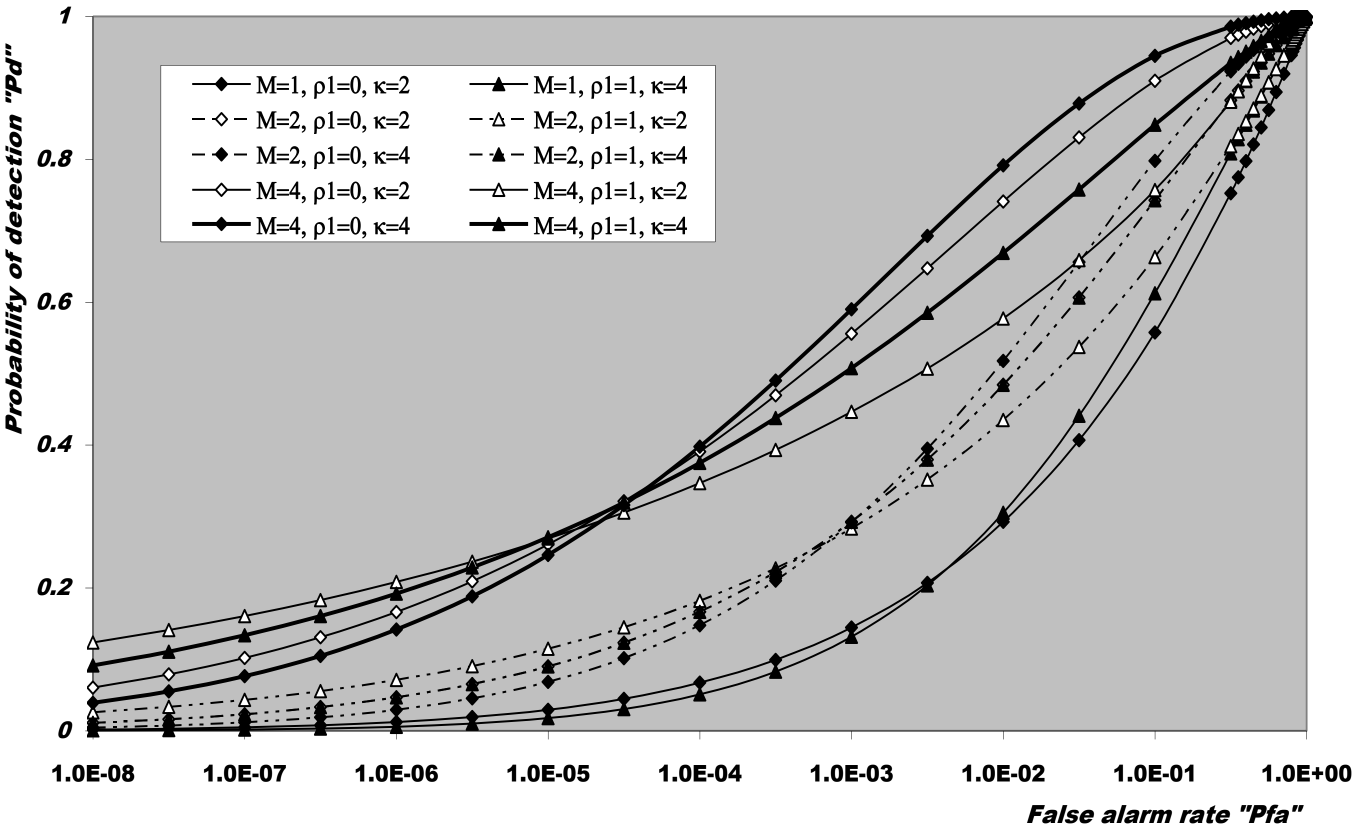

Fig.(9) M-sweeps ideal receiver operating characteristics (ROC's) of OS(21) scheme for partially-correlated $\chi 2$ targets when N=24, and Ω=5dB.

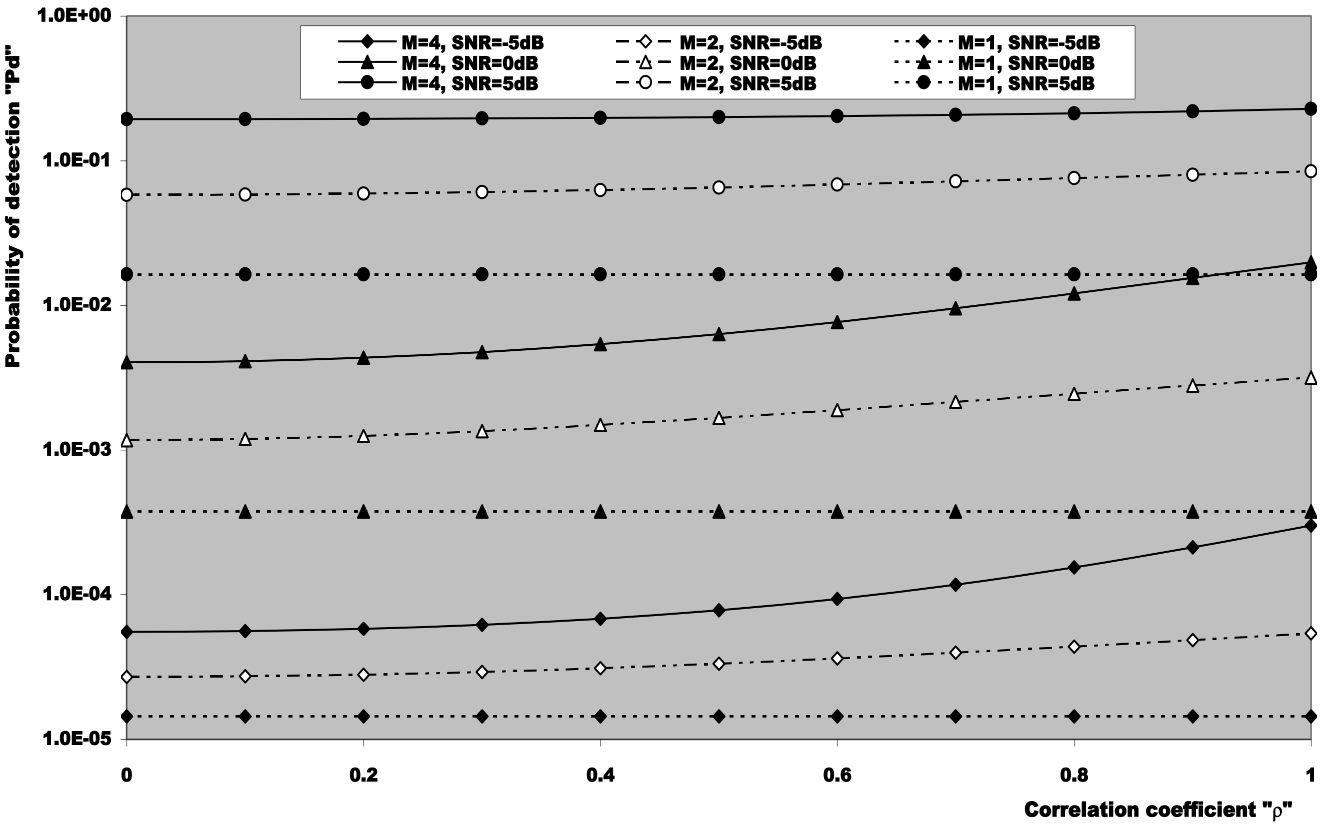

Fig.(10) M-sweeps variation of ideal detection performance of CA-CFAR processor for partially-correlated Rayleigh fluctuating targets when N=24, Pfa=1.0E-6, and SNR=-5, 0, & 5dB.

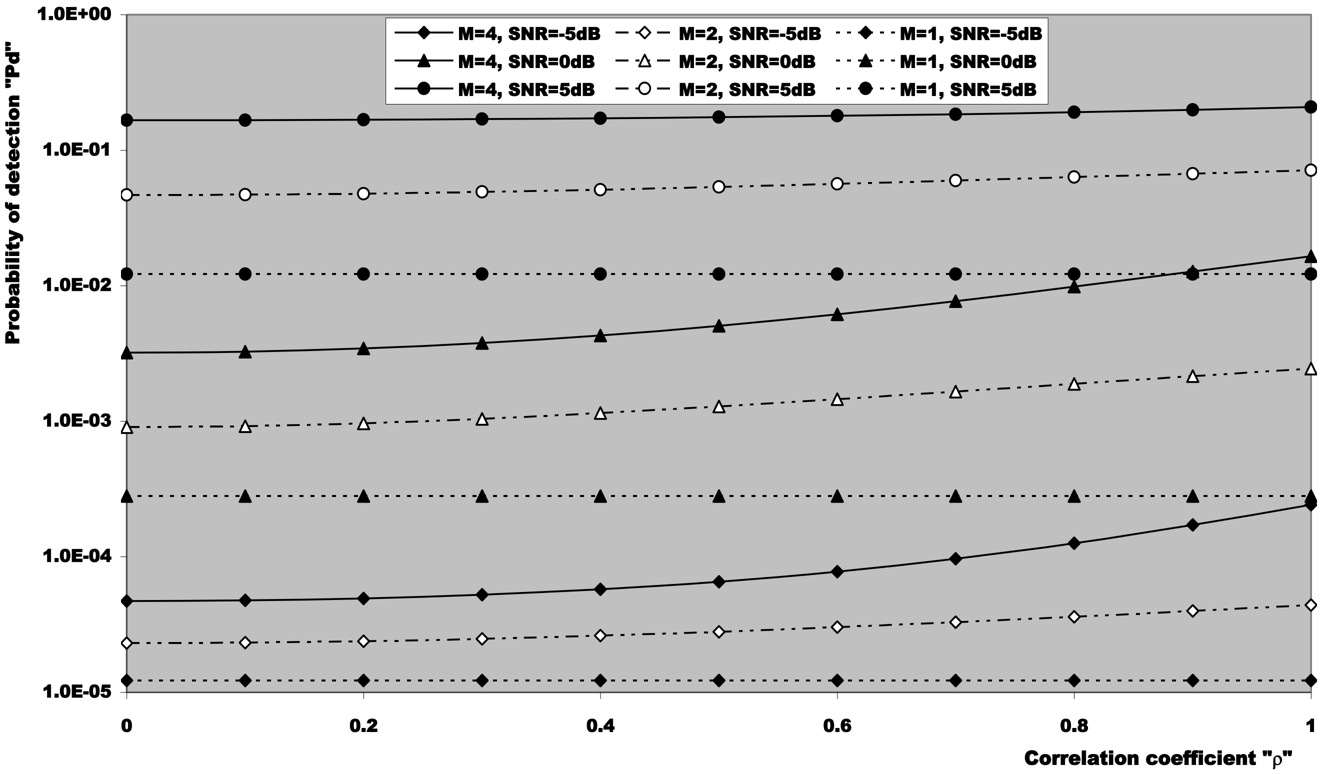

Fig.(11) M-sweeps variation of ideal detection performance of OS(21)-CFAR processor for partially-correlated Rayleigh fluctuating targets when N=24, Pfa=1.0E-6, and SNR=-5, 0, & 5dB.

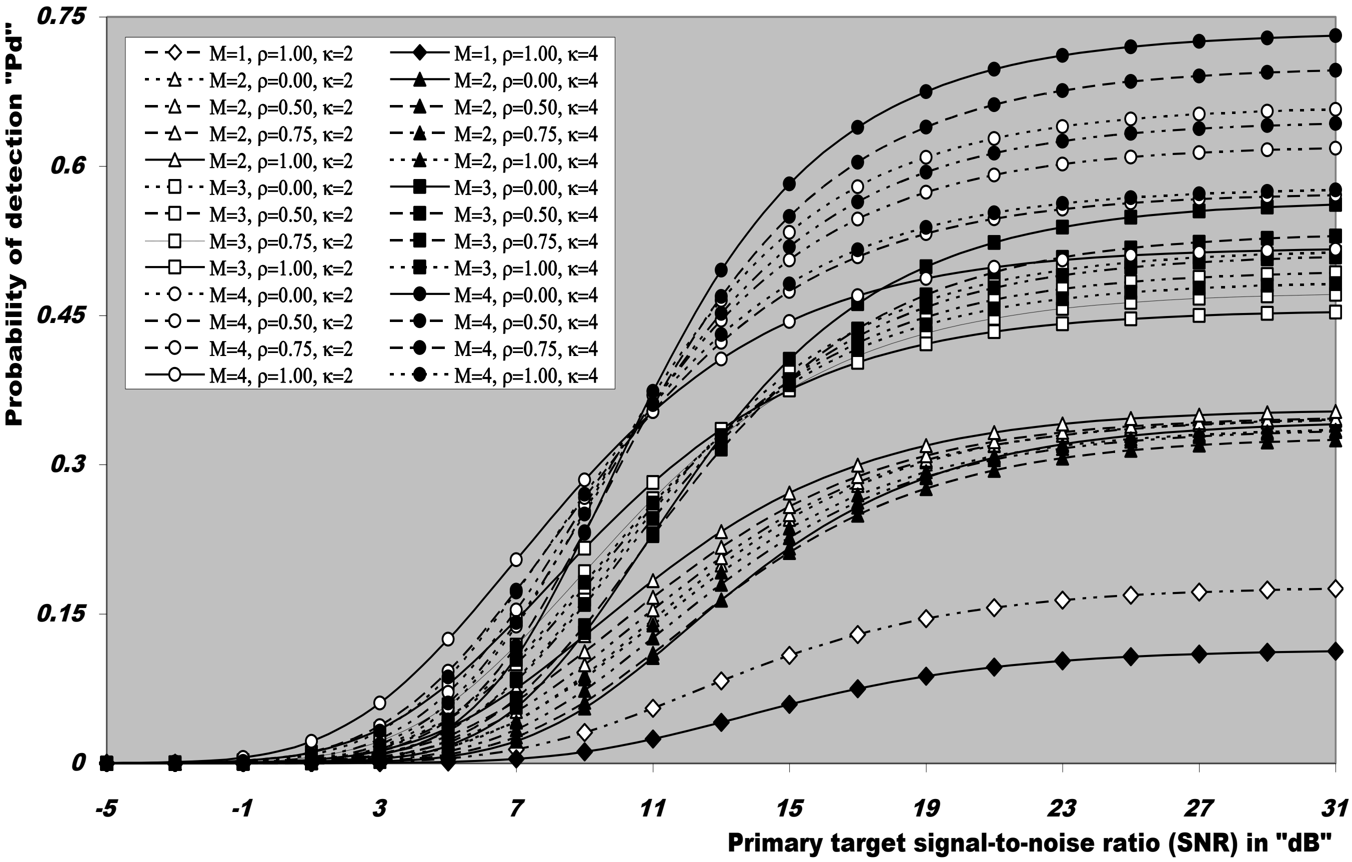

Fig.(12) Multitarget detection performance of CA scheme for partially-correlated χ2 targets with 2 & 4 degrees of freedom when N=24, r=3, and Pfa=1.0E-6.

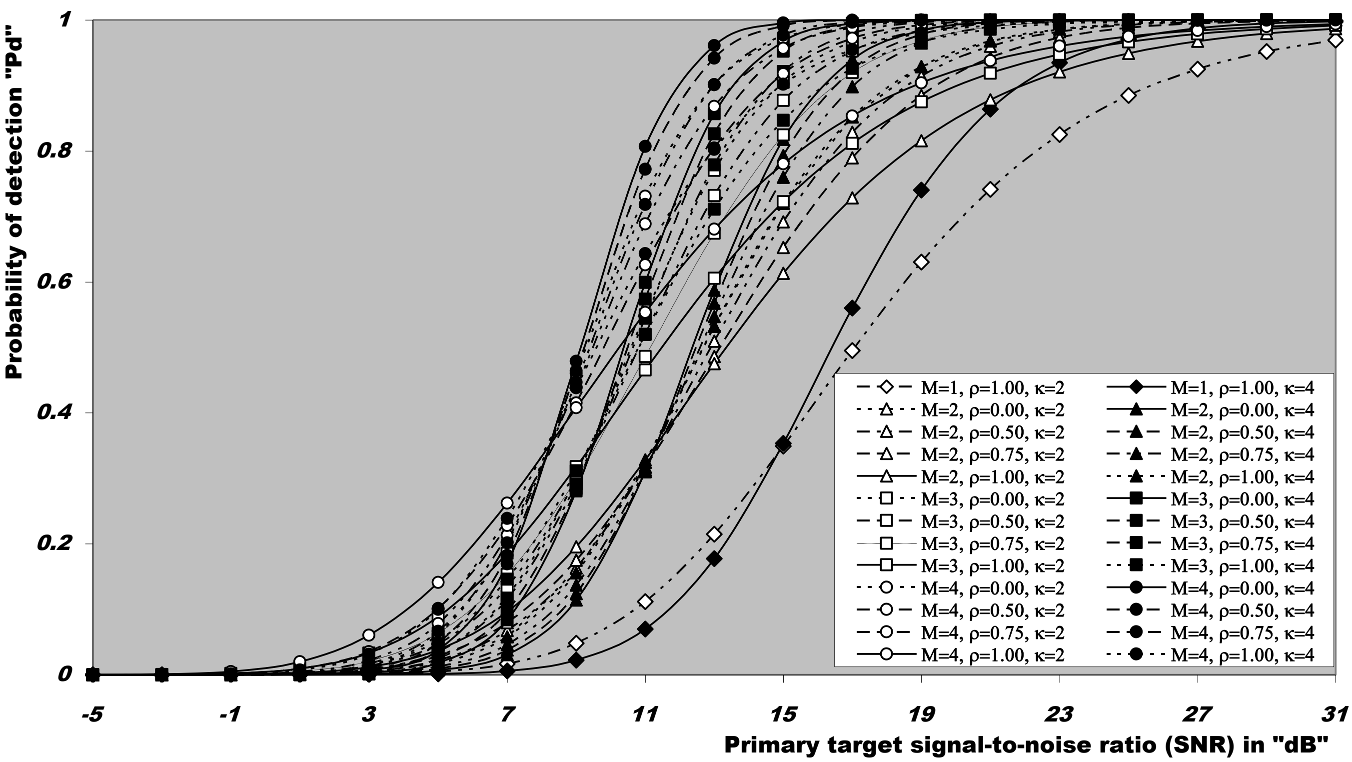

Fig.(13) Multitarget detection performance of OS(21) scheme for partially-correlated χ2 targets with 2 & 4 degrees of freedom when N=24, r=3, and Pfa=1.0E-6.

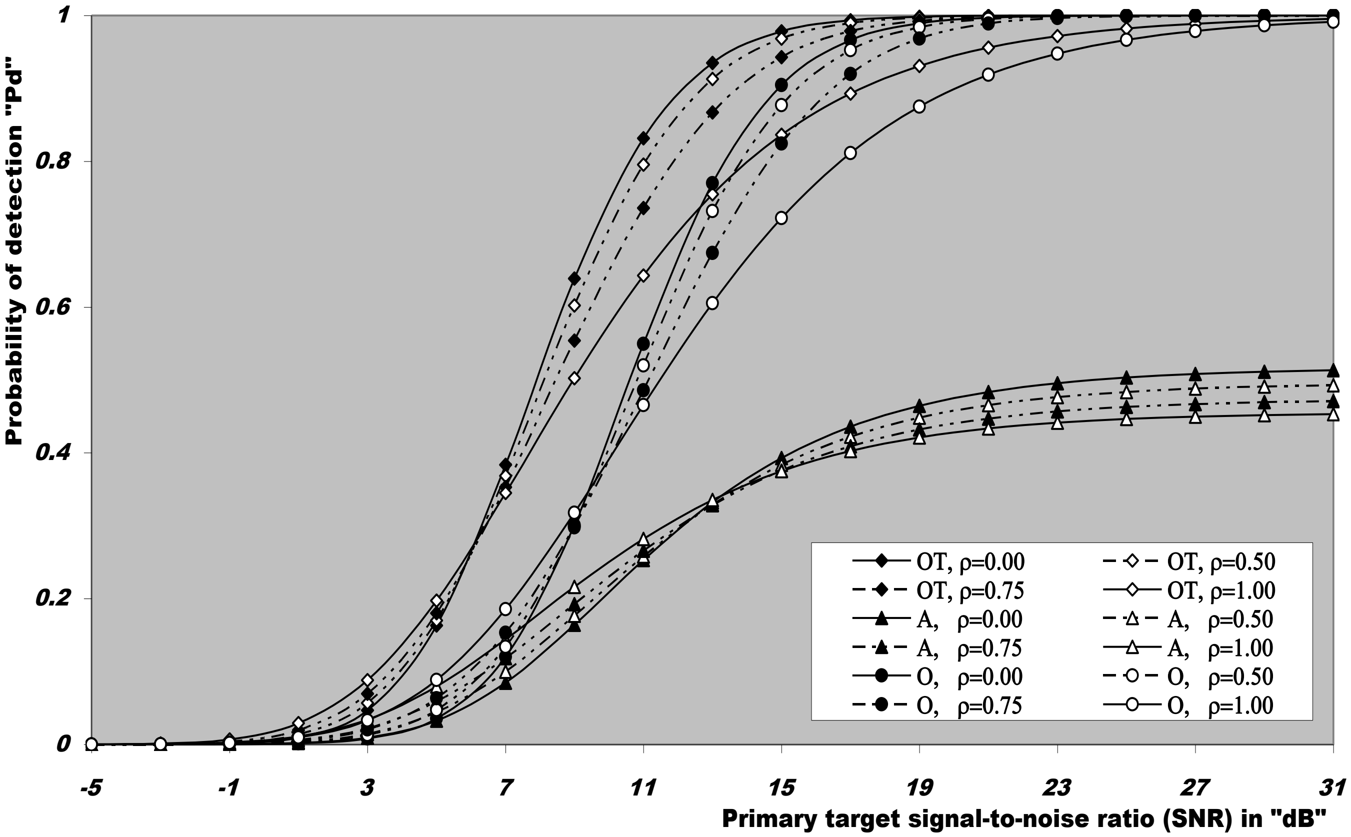

Fig.(14) M-sweeps multitarget detction performance of CFAR processors for partially-correlated $\chi 2$ targets with two degrees of freedom when N=24, M=3, r=3, Pfa=1.0E-6.

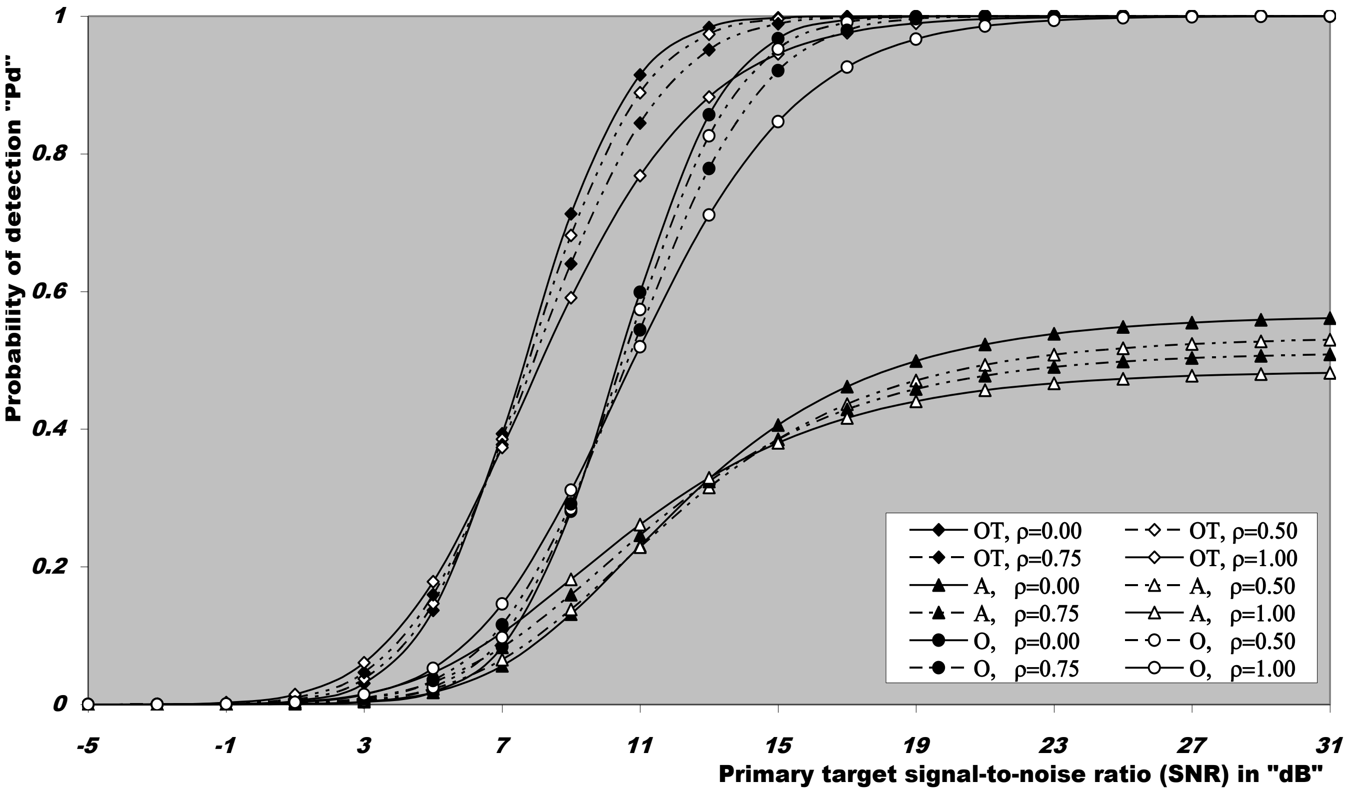

Fig.(15) M-sweeps multitarget detction performance of CFAR processors for partially-correlated $\chi 2$ targets with four degrees of freedom when N=24, M=3, r=3, Pfa=1.0E-6.

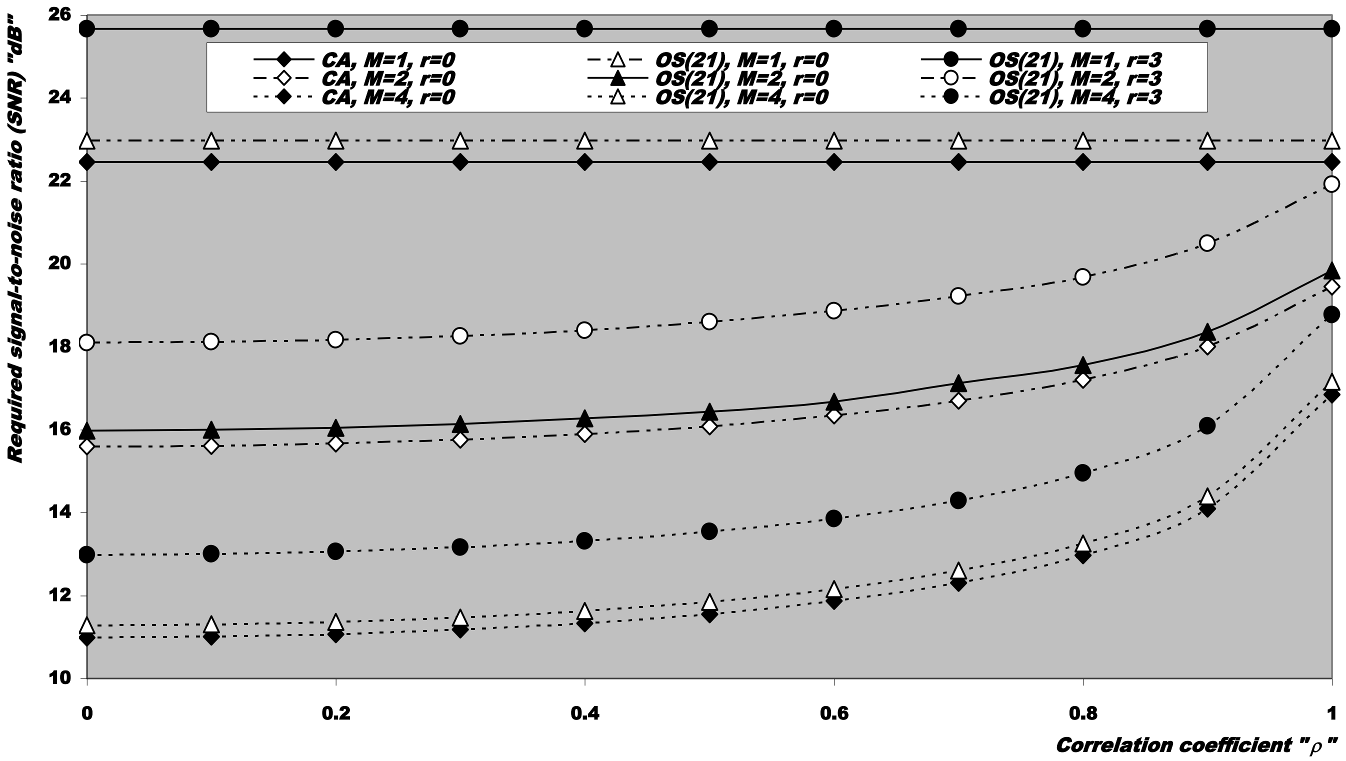

Fig.(16) M-sweeps required SNR, in the absence as well as in the presence of outlying targets, to achieve an operating point (1.0E-6, 0.9) of CFAR schemes for partially-correlated $\chi2$ targets when N=24, and κ=2.

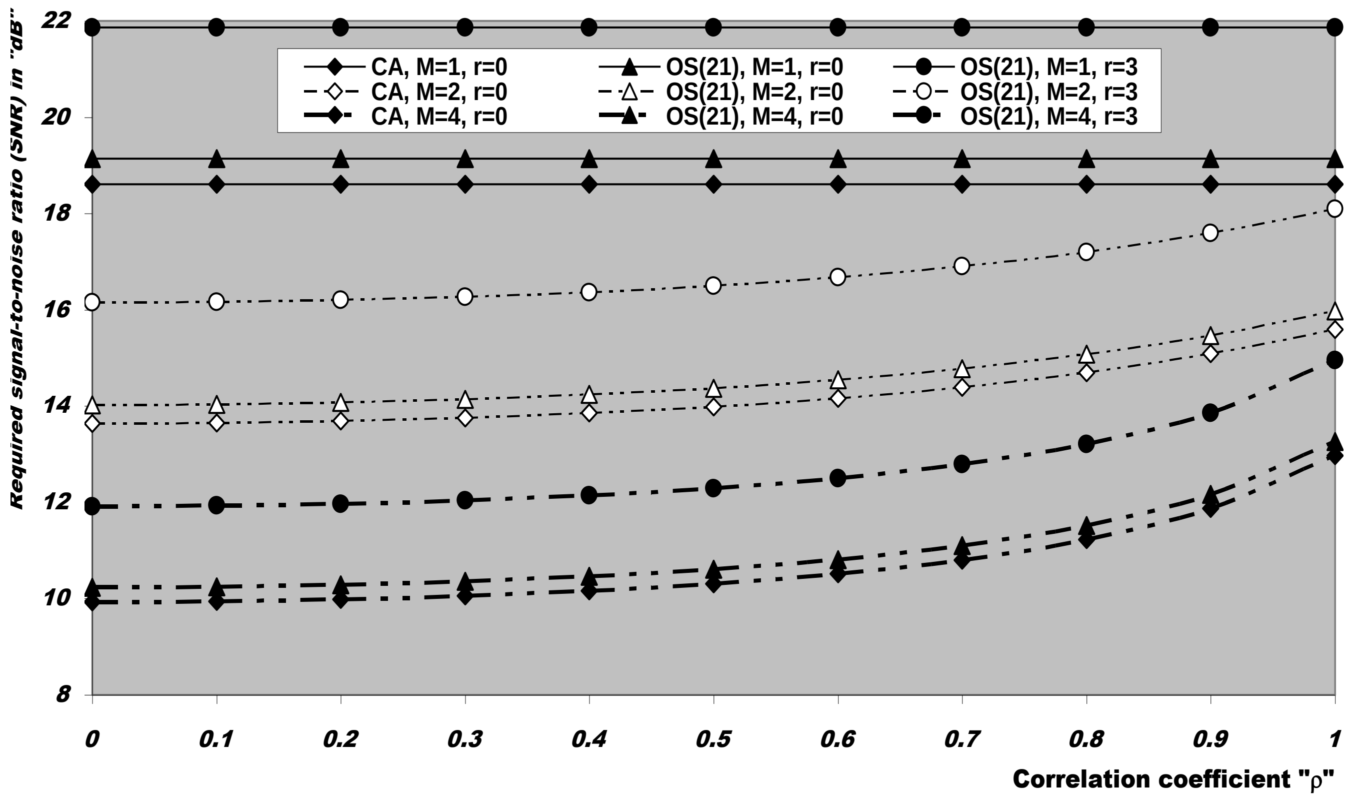

Fig.(17) M-sweeps required SNR, in the absence as well as in the presence of outlying targets, to achieve an operating point (1.0E-6, 0.90), of CFAR schemes for partially-correlated $\chi2$ targets when N=24, and κ=4.

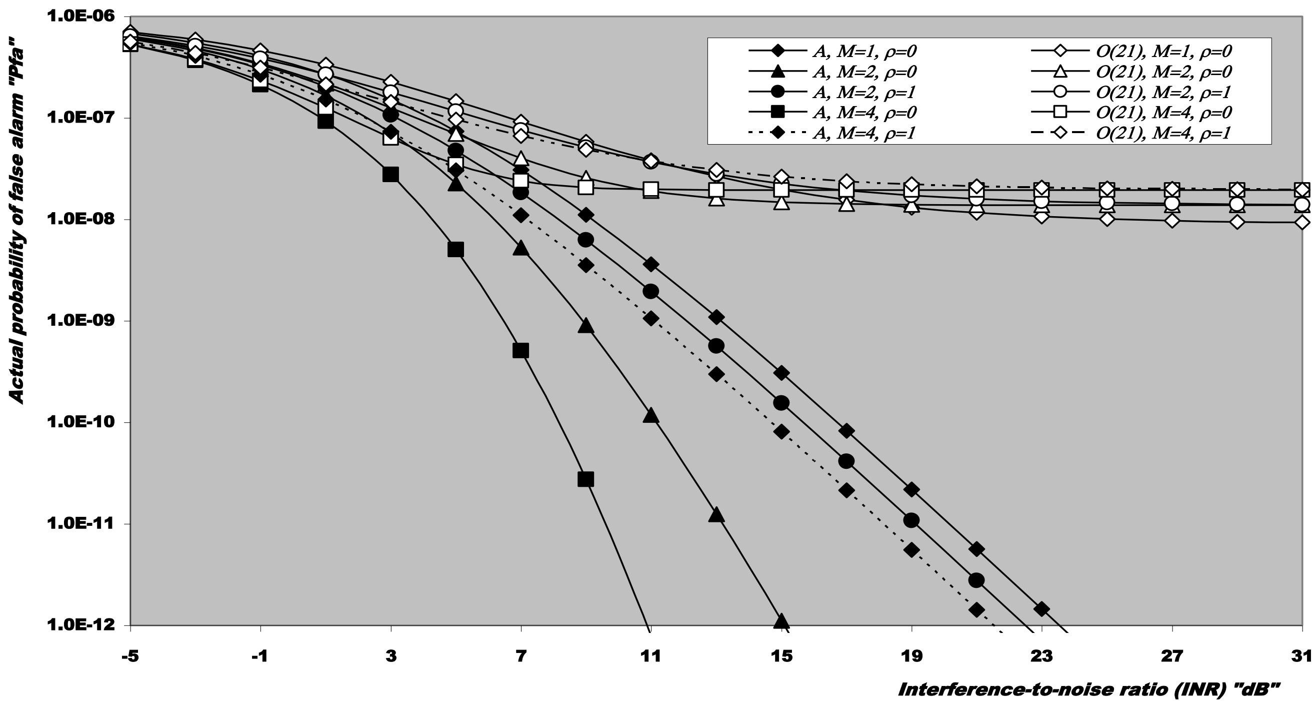

Fig.(18) M-sweeps false alarm rate performance, as a function of the interfering target strength (INR), of CFAR schemes for $\chi 2$ fluctuating targets when N=24, r=3, κ=2, and design Pfa=1.0E-6.

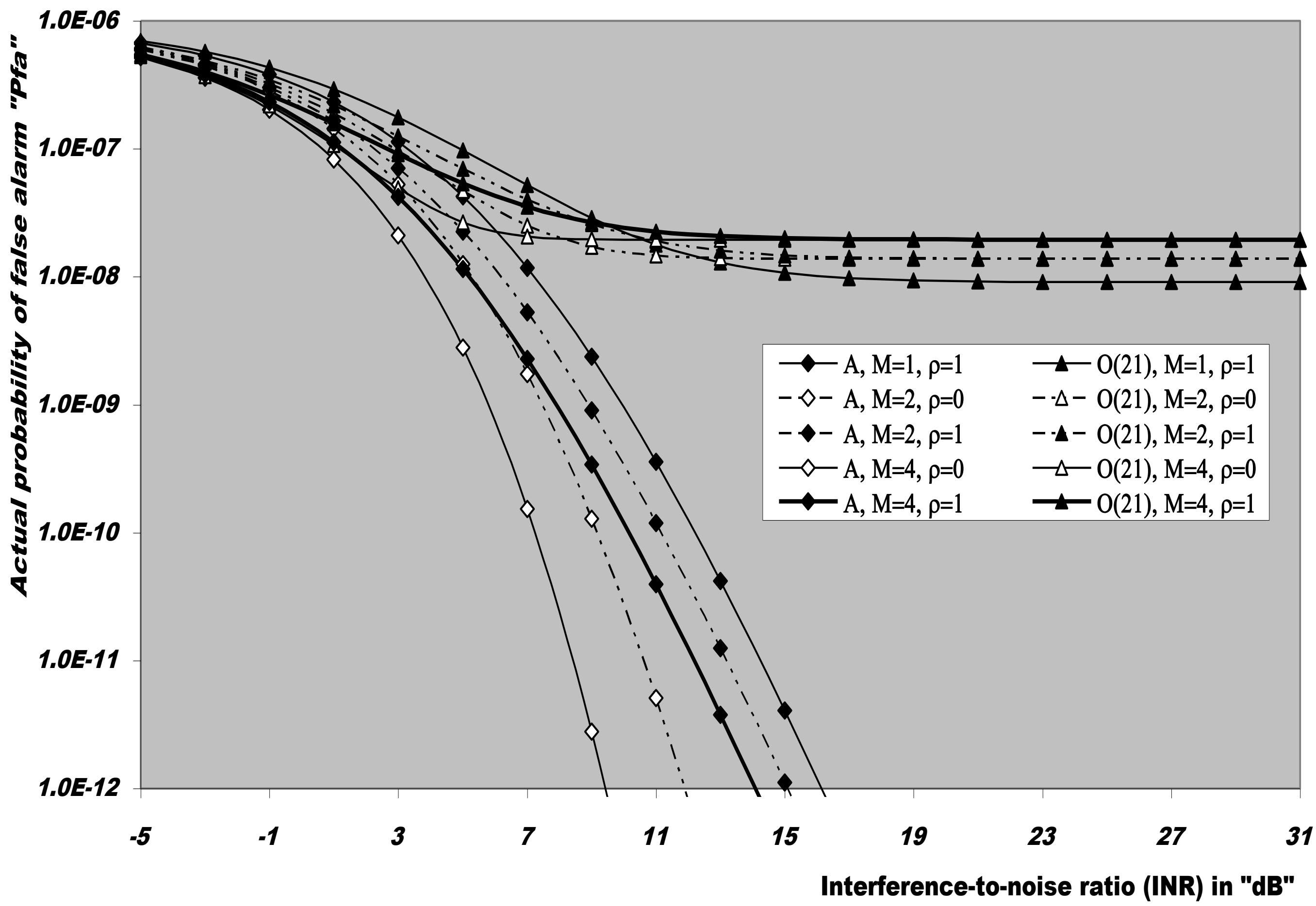

Fig.(19) M-sweeps false alarm rate performance, as a function of the interfering target strength (INR), of CFAR schemes for $\chi 2$ fluctuating targets when N=24, r=3, κ=4, and design Pfa=1.0E-6.